

Elementary Dynamics Example #51: (Rigid Body Kinetics – Impulse & Momentum – Impact #2)

Given: bar AB is initially at rest when the ball C strikes it

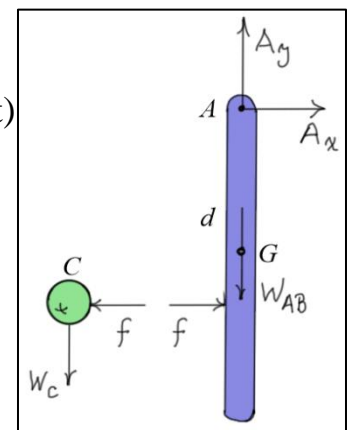
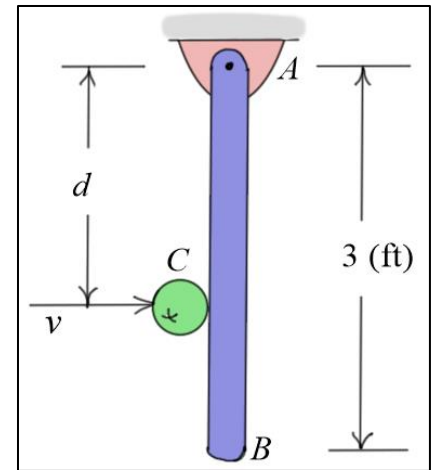
$$W_{AB} = 6 \text{ (lb)}; W_C = 1 \text{ (lb)}; e = 0.7; d = 2 \text{ (ft)}$$

just before impact, the velocity of C is $v = 50 \text{ (ft/s)}$

Find: angular velocity of AB and velocity of C just after impact

Solution: (system defined as bar AB and ball C)

The **angular impulse** on the system during the impact is **zero**. So, the **angular momentum** of the system about A is **conserved** during the impact. In the analysis that follows, the **weight forces** are assumed to be **non-impulsive**.



$$\hat{\curvearrowright} (\underline{H}_A)_1 = (\underline{H}_A)_2 \quad (\text{state 1: just before impact; state 2: just after impact})$$

$$(\underline{H}_A)_1 = \left(\frac{W_C}{g} \right) (v_C)_1 d = \frac{1 \times 50 \times 2}{32.2} \approx 3.10559 \text{ (ft-lb-s)}$$

$$(\underline{H}_A)_2 = \left(\frac{W_C}{g} \right) (v_C)_2 d + I_A (\omega_{AB})_2 \Rightarrow (\underline{H}_A)_2 = \left(\frac{2}{g} \right) (v_C)_2 + \left(\frac{18}{g} \right) (\omega_{AB})_2$$

Substituting into the **conservation of angular momentum** equation gives

$$2(v_C)_2 + 18(\omega_{AB})_2 = 3.10559 \times g = 100$$

The **impact** equation is

$$e = \frac{(v_{\hat{C}})_2 - (v_C)_2}{(v_C)_1 - (v_{\hat{C}})_1} = \frac{d(\omega_{AB})_2 - (v_C)_2}{50 - 0} \Rightarrow -(v_C)_2 + 2(\omega_{AB})_2 = 50e = 35$$

Solving the last two boxed equations **simultaneously** gives

$$\begin{aligned} (v_C)_2 &\approx -19.5455 \approx -19.5 \text{ (ft/s)} \\ (\omega_{AB})_2 &\approx 7.72727 \approx 7.73 \text{ (rad/s)} \end{aligned}$$

The ball rebounds to the **left**, and the bar **rotates counter-clockwise** at the rates shown.