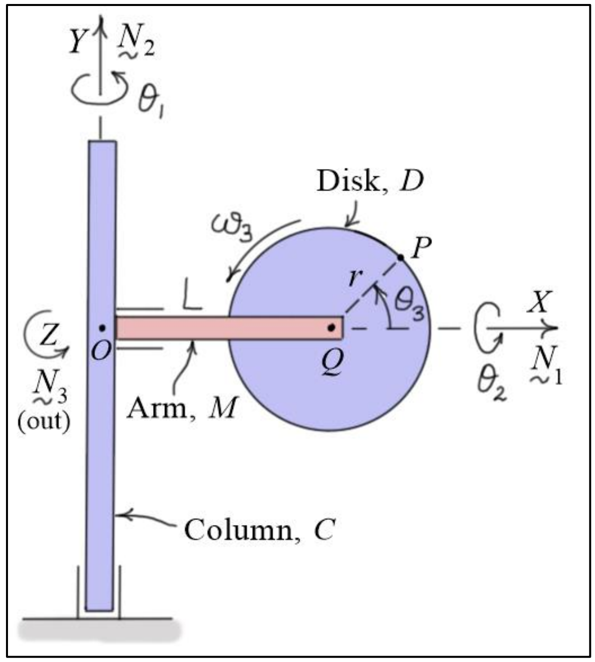
Multibody Dynamics
Exercises #1



1. The system shown has three components, a vertical column C, a horizontal arm M, and a disk D. Disk D has radius r and is positioned relative to M using angle θ_3. Arm M has length L and is positioned relative to C using angle θ_2. Column C is positioned relative to the ***fixed-frame*** XYZ using angle θ_1. The unit vectors $\underset{\sim}{N}_i \left(i=1,2,3 \right)$ are along the XYZ directions. Given the diagram, disk D is positioned relative to XYZ using a 2-1-3 body-fixed rotation sequence. Using matrix-vector notation, complete the following. In each case, find expressions for any general position where $\theta_1 \neq \theta_2 \neq \theta_3 \neq 0$. Note that in the position shown, θ_1 and θ_2 are both zero.

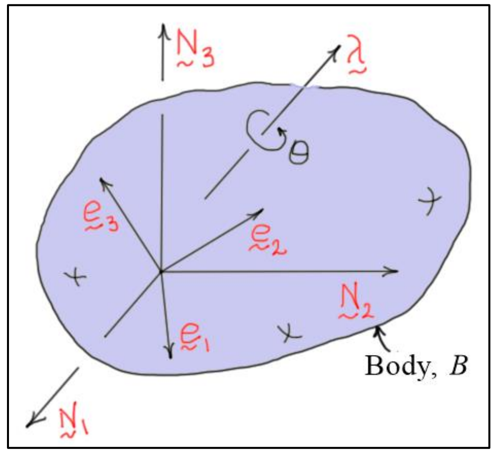
a) Find the XYZ components of $\underset{\sim}{r}_P$ the position vector of P relative to O in terms of L, r, and the three non-zero angles θ_1, θ_2, and θ_3. Express the results as a matrix-vector equation.

b) Find the XYZ components of $\underset{\sim}{v}_P$ the velocity of P relative to the fixed frame XYZ in terms of L, r, the non-zero angles θ_1, θ_2, and θ_3, and the non-zero ***body-fixed*** angular velocity components. Express the results as a matrix-vector equation. Use the concept for the relative velocity of two points fixed on a rigid body to guide the formulation.

c) Find the XYZ components of $\underset{\sim}{a}_P$ the acceleration of P relative to the fixed frame XYZ in terms of L, r, the non-zero angles θ_1, θ_2, and θ_3, and the non-zero body-fixed angular velocity and angular acceleration components. Express the results as a matrix-vector equation. Use the concept of the relative acceleration of two points fixed on a rigid body to guide the formulation.

d) Expand the matrix-vector expressions found in parts (a), (b), and (c) to find vector expressions for the position, velocity, and acceleration of P.

2. A rigid body B with unit vectors $(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3)$ is oriented relative to a base reference frame with unit vectors $(\underset{\sim}{N}_1, \underset{\sim}{N}_2, \underset{\sim}{N}_3)$ by rotating the body by a single angle $\theta = 60$ (deg) about a direction indicated by the unit vector $\underset{\sim}{\lambda} = \frac{2}{7}\underset{\sim}{N}_1 - \frac{3}{7}\underset{\sim}{N}_2 + \frac{6}{7}\underset{\sim}{N}_3$. Assuming the unit vectors of the body are initially aligned with those of the base frame, complete the following.



a) Find the four Euler parameters associated with this orientation.

b) Find the transformation matrix $[R]$ that can be used to express the body-fixed unit vectors $(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3)$ in terms of the base unit vectors $(\underset{\sim}{N}_1, \underset{\sim}{N}_2, \underset{\sim}{N}_3)$.

c) Using the transformation matrix $[R]$, express each of the unit vectors $\underset{\sim}{e}_i\ (i = 1, 2, 3)$ in terms of the unit vectors $\underset{\sim}{N}_i\ (i = 1, 2, 3)$.

3. Let the angles of the 2-1-3 body-fixed rotation sequence of problem (1) be $\theta_1 = -45$(deg), $\theta_2 = 30$ (deg), and $\theta_3 = 60$ (deg), and let the time-derivatives of the angles be $\dot{\theta}_1 = 0.5$ (rad/s), $\dot{\theta}_2 = -1.5$ (rad/s), and $\dot{\theta}_3 = 2$ (rad/s).

a) Find the four Euler parameters associated with this orientation.

b) Find the body-fixed angular velocity components associated with the rate of change of this orientation.

c) Find the time-derivatives of the Euler parameters associated with the rate of change of this orientation.