Multibody Dynamics
Exercises #1 Answers

1. a) The column rotates about the Y axis, and the disk is oriented with a body-fixed 2-1-3 rotation sequence.

$$\{r_P\} = [R_C]^T \{L'\} + [R_D]^T \{r'\}$$
$$= \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} C_1C_3 + S_1S_2S_3 & -C_1S_3 + S_1S_2C_3 & S_1C_2 \\ C_2S_3 & C_2C_3 & -S_2 \\ -S_1C_3 + C_1S_2S_3 & S_1S_3 + C_1S_2C_3 & C_1C_2 \end{bmatrix} \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}$$

b) $\omega'_{Di}\ (i = 1,2,3)$ are the body-fixed components of the angular velocity of the disk.

$$\{{}^R v_P\} = [R_C]^T [\tilde{\omega}'_C] \{L'\} + [R_D]^T [\tilde{\omega}'_D] \{r'\} = \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix}$$
$$+ \begin{bmatrix} C_1C_3 + S_1S_2S_3 & -C_1S_3 + S_1S_2C_3 & S_1C_2 \\ C_2S_3 & C_2C_3 & -S_2 \\ -S_1C_3 + C_1S_2S_3 & S_1S_3 + C_1S_2C_3 & C_1C_2 \end{bmatrix} \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \omega'_{D1} \\ \omega'_{D2} \\ \omega'_{D3} \end{Bmatrix} = \begin{Bmatrix} \dot{\theta}_1 C_2 S_3 + \dot{\theta}_2 C_3 \\ \dot{\theta}_1 C_2 C_3 - \dot{\theta}_2 S_3 \\ -\dot{\theta}_1 S_2 + \dot{\theta}_3 \end{Bmatrix}$$ (for a 2-1-3 body-fixed rotation sequence)

c) $\alpha'_{Di}\ (i = 1,2,3)$ are the body-fixed components of the angular acceleration of the disk.

$$\{{}^R a_P\} = [R_C]^T \left([\tilde{\alpha}'_C] + [\tilde{\omega}'_C][\tilde{\omega}'_C] \right) \{L'\} + [R_D]^T \left([\tilde{\alpha}'_D] + [\tilde{\omega}'_D][\tilde{\omega}'_D] \right) \{r'\}$$
$$= \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & \ddot{\theta}_1 \\ 0 & 0 & 0 \\ -\ddot{\theta}_1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix} +$$
$$\begin{bmatrix} C_1C_3 + S_1S_2S_3 & -C_1S_3 + S_1S_2C_3 & S_1C_2 \\ C_2S_3 & C_2C_3 & -S_2 \\ -S_1C_3 + C_1S_2S_3 & S_1S_3 + C_1S_2C_3 & C_1C_2 \end{bmatrix} \left(\begin{bmatrix} 0 & -\alpha'_{D3} & \alpha'_{D2} \\ \alpha'_{D3} & 0 & -\alpha'_{D1} \\ -\alpha'_{D2} & \alpha'_{D1} & 0 \end{bmatrix} + \right.$$
$$\left. \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \right) \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}$$

d) The column vectors below show the $\underset{\sim}{N}_1$, $\underset{\sim}{N}_2$, and $\underset{\sim}{N}_3$ components of the position, velocity, and acceleration vectors of *P*.

$$\left\{r_P\right\} = \begin{Bmatrix} LC_1 + r\left(C_1C_3 + S_1S_2S_3\right) \\ r\,C_2S_3 \\ -LS_1 + r\left(-S_1C_3 + C_1S_2S_3\right) \end{Bmatrix}$$

$$\left\{{}^R v_P\right\} = \begin{Bmatrix} -LS_1\dot{\theta}_1 + r\left(-C_1S_3 + S_1S_2C_3\right)\omega'_{D3} - r\,S_1C_2\omega'_{D2} \\ r\left(C_2C_3\omega'_{D3} + S_2\omega'_{D2}\right) \\ -LC_1\dot{\theta}_1 + r\left(S_1S_3 + C_1S_2C_3\right)\omega'_{D3} - r\,C_1C_2\omega'_{D2} \end{Bmatrix}$$

$$\left\{{}^R a_P\right\} = L\begin{Bmatrix} -\left(C_1\dot{\theta}_1^2 + S_1\ddot{\theta}_1\right) \\ 0 \\ \left(S_1\dot{\theta}_1^2 - C_1\ddot{\theta}_1\right) \end{Bmatrix} + r\begin{Bmatrix} -\left(C_1C_3 + S_1S_2S_3\right)\left(\omega'^2_{D2} + \omega'^2_{D3}\right) + \left(-C_1S_3 + S_1S_2C_3\right)\left(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}\right) + S_1C_2\left(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}\right) \\ -C_2S_3\left(\omega'^2_{D2} + \omega'^2_{D3}\right) + C_2C_3\left(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}\right) - S_2\left(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}\right) \\ \left(S_1C_3 - C_1S_2S_3\right)\left(\omega'^2_{D2} + \omega'^2_{D3}\right) + \left(S_1S_3 + C_1S_2C_3\right)\left(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}\right) + C_1C_2\left(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}\right) \end{Bmatrix}$$

2. a) $\varepsilon_1 = 0.142857;\ \varepsilon_2 = -0.214286;\ \varepsilon_3 = 0.428571;\ \varepsilon_4 = 0.866025$

   b) $$[R] = \begin{bmatrix} 0.540816 & 0.681083 & 0.493603 \\ -0.803532 & 0.591837 & 0.063762 \\ -0.248705 & -0.431109 & 0.867347 \end{bmatrix}$$

   c) $\underset{\sim}{e}_1 = 0.540816\,\underset{\sim}{N}_1 + 0.681083\,\underset{\sim}{N}_2 + 0.493603\,\underset{\sim}{N}_3$

   $\underset{\sim}{e}_2 = -0.803532\,\underset{\sim}{N}_1 + 0.591837\,\underset{\sim}{N}_2 + 0.063762\,\underset{\sim}{N}_3$

   $\underset{\sim}{e}_3 = -0.248705\,\underset{\sim}{N}_1 - 0.431109\,\underset{\sim}{N}_2 + 0.867347\,\underset{\sim}{N}_3$

3. a) $\varepsilon_1 = 0.022260;\ \varepsilon_2 = -0.439680;\ \varepsilon_3 = 0.531976;\ \varepsilon_4 = 0.723317$

   b) $\omega'_1 = -0.375$ (rad/s); $\omega'_2 = 1.51554$ (rad/s); $\omega'_3 = 1.75$ (rad/s)

   c) $\dot{\varepsilon}_1 = 0.652214;\ \dot{\varepsilon}_2 = 0.667333;\ \dot{\varepsilon}_3 = 698475;\ \dot{\varepsilon}_4 = -0.128128$