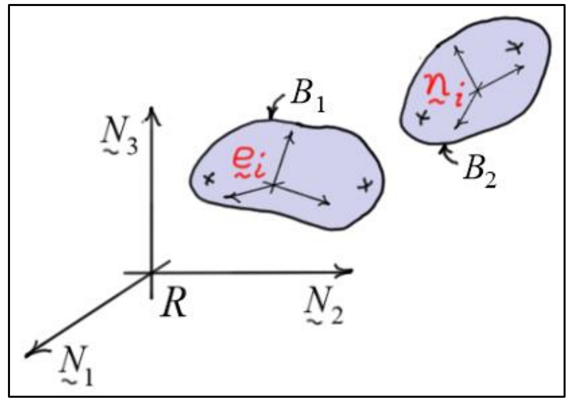
Multibody Dynamics
Exercises #2

A two-body system is shown in each of the two figures. The orientation of each of the bodies is specified relative to the inertial frame R (***absolute angles***) using a 3-2-1 body-fixed rotation sequence. The angles for B_1 and B_2 are $\theta_{1i}\ (i=1,2,3)$ and $\theta_{2i}\ (i=1,2,3)$, respectively. The position of the bodies is to be specified using ***relative coordinates***. The position of body B_1 is given relative to R, and the position of B_2 is given relative to body B_1 as shown in the lower figure. The vectors $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$ represent translation vectors for each body written in terms of the base reference frame.



$$\boxed{\underset{\sim}{s}_1=\sum_{i=1}^{3}x_{1i}\,\underset{\sim}{N}_i} \quad \text{and} \quad \boxed{\underset{\sim}{s}_2=\sum_{i=1}^{3}x_{2i}\,\underset{\sim}{e}_i}$$

The position vectors $\underset{\sim}{r}_1$ and $\underset{\sim}{q}_2$ are fixed in B_1 so their components are given in B_1, and the position vector $\underset{\sim}{r}_2$ is fixed in B_2 so its components are given in B_2. Complete the following:

a) Find the ***body-fixed components*** of the angular velocities of the bodies. Express the results in matrix-vector form. Then, identify the partial angular velocity matrices of each body associated with $\dot{\theta}_{1i}\ (i=1,2,3)$ and $\dot{\theta}_{2i}\ (i=1,2,3)$.

b) Find the ***inertial components*** of the mass-center position vectors of the bodies. Express the results in matrix-vector form.

c) Find the ***inertial components*** of the mass-center velocities of the bodies. Express the results in matrix-vector form using the body-fixed angular velocity components. Then, identify the partial velocity matrices of each body associated with $\dot{x}_{1i}\ (i=1,2,3)$, $\dot{x}_{2i}\ (i=1,2,3)$, $\dot{\theta}_{1i}\ (i=1,2,3)$, and $\dot{\theta}_{2i}\ (i=1,2,3)$.