

Multibody Dynamics

Exercises #2 Answers

a) $\{\omega'_{B_i}\} = [\omega'_{B_i, \dot{\theta}_i}] \{\dot{\theta}_i\} \quad (i=1,2) \quad \{\theta_i\} \triangleq [\theta_{i1} \quad \theta_{i2} \quad \theta_{i3}]^T \quad (i=1,2)$ (the angles for body “i”)

$$[\omega'_{B_i, \dot{\theta}_i}] = \begin{bmatrix} -S_{i2} & 0 & 1 \\ C_{i2}S_{i3} & C_{i3} & 0 \\ C_{i2}C_{i3} & -S_{i3} & 0 \end{bmatrix} \quad (i=1,2)$$

$$[\omega'_{B_i, \dot{\theta}_i}] = [0]_{3 \times 3} \quad (i \neq j)$$

b) $\{p_{G_1}\} = \{s_1\} + [R_{B_1}]^T \{r'_1\}$

$$\{p_{G_2}\} = \{s_1\} + [R_{B_1}]^T (\{q'_2\} + \{s'_2\}) + [R_{B_2}]^T \{r'_2\}$$

$$[R_{B_i}] = \begin{bmatrix} C_{i1}C_{i2} & S_{i1}C_{i2} & -S_{i2} \\ -S_{i1}C_{i3} + C_{i1}S_{i2}S_{i3} & C_{i1}C_{i3} + S_{i1}S_{i2}S_{i3} & C_{i2}S_{i3} \\ S_{i1}S_{i3} + C_{i1}S_{i2}C_{i3} & -C_{i1}S_{i3} + S_{i1}S_{i2}C_{i3} & C_{i2}C_{i3} \end{bmatrix}$$

c) $\{v_{G_i}\} = [v_{G_i, \dot{s}_1}] \{\dot{s}_1\} + [v_{G_i, \dot{\theta}_1}] \{\dot{\theta}_1\} + [v_{G_i, \dot{s}'_2}] \{\dot{s}'_2\} + [v_{G_i, \dot{\theta}_2}] \{\dot{\theta}_2\} \quad (i=1,2)$

where

$$[v_{G_1, \dot{s}_1}] = [v_{G_2, \dot{s}_1}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[v_{G_1, \dot{s}'_2}] = [0]_{3 \times 3}$$

$$[v_{G_1, \dot{\theta}_1}] = -[R_{B_1}]^T [\tilde{r}'_1] [\omega'_{B_1, \dot{\theta}_1}]$$

$$[v_{G_1, \dot{\theta}_{B_2}}] = [0]_{3 \times 3}$$

$$[v_{G_2, \dot{s}'_2}] = [R_{B_1}]^T$$

$$[v_{G_2, \dot{\theta}_1}] = -[R_{B_1}]^T ([\tilde{q}'_2] + [\tilde{s}'_2]) [\omega'_{B_1, \dot{\theta}_1}]$$

$$[v_{G_2, \dot{\theta}_2}] = -[R_{B_2}]^T [\tilde{r}'_2] [\omega'_{B_2, \dot{\theta}_2}]$$