

Multibody Dynamics

Exercises #3 Answers

$$1. \{a_{G_3}\} = \{\ddot{s}_1\} + \sum_{i=2}^3 [v_{G_3, s'_i}] \{\ddot{s}'_i\} + \sum_{i=2}^3 [\dot{v}_{G_3, s'_i}] \{\dot{s}'_i\} + \sum_{i=1}^3 [v_{G_3, \dot{\theta}_i}] \{\ddot{\theta}_i\} + \sum_{i=1}^3 [\dot{v}_{G_3, \dot{\theta}_i}] \{\dot{\theta}_i\}$$

Here, the partial velocities are as given in Exercises #2. The derivatives of the partial velocities are

$$\begin{aligned} [\dot{v}_{G_2, s'_2}] &= [\dot{R}_{B_1}]^T = [R_{B_1}]^T [\dot{\omega}'_{B_1}] \\ [\dot{v}_{G_2, \dot{\theta}_1}] &= -[R_{B_1}]^T [\dot{\omega}'_{B_1}] ([\tilde{q}'_2] + [\tilde{s}'_2]) [\omega'_{B_1, \dot{\theta}_1}] - [R_{B_1}]^T [\dot{s}'_2] [\omega'_{B_1, \dot{\theta}_1}] \\ &\quad - [R_{B_1}]^T ([\tilde{q}'_2] + [\tilde{s}'_2]) [\dot{\omega}'_{B_1, \dot{\theta}_1}] \\ [\dot{v}_{G_2, \dot{\theta}_2}] &= -[R_{B_2}]^T [\dot{\omega}'_{B_2}] [\tilde{r}'_2] [\omega'_{B_2, \dot{\theta}_2}] - [R_{B_2}]^T [\tilde{r}'_2] [\dot{\omega}'_{B_2, \dot{\theta}_2}] \end{aligned}$$

$$2. \{a_{G_3}\} = \{\ddot{s}_1\} + \sum_{i=2}^3 [v_{G_3, s'_i}] \{\ddot{s}'_i\} + \sum_{i=2}^3 [\dot{v}_{G_3, s'_i}] \{\dot{s}'_i\} + \sum_{i=1}^3 [v_{G_3, \dot{\theta}_i}] \{\ddot{\theta}_i\} + \sum_{i=1}^3 [\dot{v}_{G_3, \dot{\theta}_i}] \{\dot{\theta}_i\}$$

where the partial velocities are as given in Exercises #2. The derivatives of the partial velocities are

$$\begin{aligned} [\dot{v}_{G_3, s'_2}] &= [\dot{R}_{B_1}]^T = [R_{B_1}]^T [\dot{\omega}'_{B_1}] \\ [\dot{v}_{G_3, s'_3}] &= [\dot{R}_{B_2}]^T = [R_{B_2}]^T [\dot{\omega}'_{B_2}] \\ [\dot{v}_{G_3, \dot{\theta}_1}] &= -[R_{B_1}]^T [\dot{\omega}'_{B_1}] ([\tilde{q}'_2] + [\tilde{s}'_2]) [\omega'_{B_1, \dot{\theta}_1}] - [R_{B_1}]^T [\dot{s}'_2] [\omega'_{B_1, \dot{\theta}_1}] \\ &\quad - [R_{B_1}]^T ([\tilde{q}'_2] + [\tilde{s}'_2]) [\dot{\omega}'_{B_1, \dot{\theta}_1}] \\ [\dot{v}_{G_3, \dot{\theta}_2}] &= -[R_{B_2}]^T [\dot{\omega}'_{B_2}] ([\tilde{q}'_3] + [\tilde{s}'_3]) [\omega'_{B_2, \dot{\theta}_2}] - [R_{B_2}]^T [\dot{s}'_3] [\omega'_{B_2, \dot{\theta}_2}] \\ &\quad - [R_{B_2}]^T ([\tilde{q}'_3] + [\tilde{s}'_3]) [\dot{\omega}'_{B_2, \dot{\theta}_2}] \\ [\dot{v}_{G_3, \dot{\theta}_3}] &= -[R_{B_3}]^T [\dot{\omega}'_{B_3}] [\tilde{r}'_3] [\omega'_{B_3, \dot{\theta}_3}] - [R_{B_3}]^T [\tilde{r}'_3] [\dot{\omega}'_{B_3, \dot{\theta}_3}] \end{aligned}$$