Multibody Dynamics
Exercises #4 Answers

1. a) Relative coordinates:

Translation constraints (6 equations)

$$\boxed{\{s_1\} = \{0\}_{3\times 1}} \quad \text{and} \quad \boxed{\{s_2'\} = \{0\}_{3\times 1}}$$

Rotation constraints (3 equations) with $\boxed{{}^R\underset{\sim}{\omega}_{L_1} = \underset{\sim}{\hat{\omega}}_{L_1} = \sum_{i=1}^{3} \hat{\omega}_{1i}' \, \underset{\sim}{e}_{1i}}$, $\boxed{{}^{B_1}\underset{\sim}{\omega}_{L_2} = \underset{\sim}{\hat{\omega}}_{L_2} = \sum_{i=1}^{3} \hat{\omega}_{2i}' \, \underset{\sim}{e}_{2i}}$

$$\boxed{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \left[R_{L_1} \right]^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}_{11}' \\ \hat{\omega}_{12}' \\ \hat{\omega}_{13}' \end{Bmatrix}} \quad \text{and} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}_{21}' \\ \hat{\omega}_{22}' \\ \hat{\omega}_{23}' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

b) Absolute coordinates:

Translation constraints (6 equations)

$$\boxed{\left\{ p_{G_1} \right\} + \tfrac{1}{2}\ell_1 \left[R_{L_1} \right]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}} \quad \boxed{\left\{ p_{G_2} \right\} - \left\{ p_{G_1} \right\} + \tfrac{1}{2}\ell_1 \left[R_{L_1} \right]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + \tfrac{1}{2}\ell_2 \left[R_{L_2} \right]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}}$$

Rotation constraints (3 equations) with $\boxed{{}^R\underset{\sim}{\omega}_{L_1} = \sum_{i=1}^{3} \omega_{1i}' \, \underset{\sim}{e}_{1i}}$ and $\boxed{{}^R\underset{\sim}{\omega}_{L_2} = \sum_{i=1}^{3} \omega_{2i}' \, \underset{\sim}{e}_{2i}}$

$$\boxed{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \left[R_{L_1} \right]^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \omega_{11}' \\ \omega_{12}' \\ \omega_{13}' \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \left(\begin{Bmatrix} \omega_{21}' \\ \omega_{22}' \\ \omega_{23}' \end{Bmatrix} - \left[R_{L_2} \right] \left[R_{L_1} \right]^T \begin{Bmatrix} \omega_{11}' \\ \omega_{12}' \\ \omega_{13}' \end{Bmatrix} \right) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

2. a) Relative coordinates:

Translation constraints (8 equations)

$$\boxed{\{s_1\} = \{0\}_{3\times 1}} \quad \boxed{\{s_2'\} = \{0\}_{3\times 1}} \quad \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} s_{31}' \\ s_{32}' \\ s_{33}' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

Rotation constraints (6 eq.): $\boxed{{}^R\underset{\sim}{\omega}_B = \underset{\sim}{\hat{\omega}}_B = \sum_{i=1}^{3} \hat{\omega}_{Bi}' \, \underset{\sim}{e}_{B_i}}$ $\boxed{{}^B\underset{\sim}{\omega}_M = \underset{\sim}{\hat{\omega}}_M = \sum_{i=1}^{3} \hat{\omega}_{Mi}' \, \underset{\sim}{e}_{M_i}}$ $\boxed{{}^M\underset{\sim}{\omega}_E = \underset{\sim}{\hat{\omega}}_E = \sum_{i=1}^{3} \hat{\omega}_{Ei}' \, \underset{\sim}{e}_{E_i}}$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \hat{\omega}_{B1}' \\ \hat{\omega}_{B2}' \\ \hat{\omega}_{B3}' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}_{M1}' \\ \hat{\omega}_{M2}' \\ \hat{\omega}_{M3}' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \hat{\omega}_{E1}' \\ \hat{\omega}_{E2}' \\ \hat{\omega}_{E3}' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

b) Absolute Coordinates:

Translation constraints (8 equations)

$$\left\{p_{G_B}\right\} - \begin{Bmatrix}0\\ \ell_1\\ 0\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

$$\left\{p_{G_M}\right\} - \left\{p_{G_B}\right\} - \begin{Bmatrix}0\\ \ell_2\\ 0\end{Bmatrix} - \left[R_M\right]^T \begin{Bmatrix}\ell_3\\ 0\\ 0\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\left(\left[R_M\right]\left\{p_{G_E}\right\} - \left[R_M\right]\left\{p_{G_M}\right\} - \left(\ell_4 + \ell_5\right)\begin{Bmatrix}1\\0\\0\end{Bmatrix}\right) = \begin{Bmatrix}0\\0\end{Bmatrix}$$

Rotation constraints (6 equations):

$${}^R\underset{\sim}{\omega}_B = \sum_{i=1}^{3} \omega'_{Bi}\, \underset{\sim}{e}_{B_i} \qquad {}^R\underset{\sim}{\omega}_M = \sum_{i=1}^{3} \omega'_{Mi}\, \underset{\sim}{e}_{M_i} \qquad {}^R\underset{\sim}{\omega}_E = \sum_{i=1}^{3} \omega'_{Ei}\, \underset{\sim}{e}_{E_i}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\omega'_{B1}\\ \omega'_{B2}\\ \omega'_{B3}\end{Bmatrix} = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\left(\begin{Bmatrix}\omega'_{M1}\\ \omega'_{M2}\\ \omega'_{M3}\end{Bmatrix} - \left[R_M\right]\left[R_B\right]^T \begin{Bmatrix}\omega'_{B1}\\ \omega'_{B2}\\ \omega'_{B3}\end{Bmatrix}\right) = \begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\left(\begin{Bmatrix}\omega'_{E1}\\ \omega'_{E2}\\ \omega'_{E3}\end{Bmatrix} - \left[R_E\right]\left[R_M\right]^T \begin{Bmatrix}\omega'_{M1}\\ \omega'_{M2}\\ \omega'_{M3}\end{Bmatrix}\right) = \begin{Bmatrix}0\\0\end{Bmatrix}$$