

Multibody Dynamics

Exercises #5 Answers

1.

$$[I_G] = ma^2 \begin{bmatrix} \frac{47}{12} & 1 & -\frac{3}{2} \\ 1 & \frac{71}{12} & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & \frac{10}{3} \end{bmatrix} \approx ma^2 \begin{bmatrix} 3.91667 & 1.00000 & -1.50000 \\ 1.00000 & 5.91667 & 0.500000 \\ -1.50000 & 0.500000 & 3.33333 \end{bmatrix}$$

$$[M] \approx \begin{bmatrix} 0.642082 & -0.648393 & -0.409045 \\ -0.246360 & 0.330748 & -0.910995 \\ 0.725974 & 0.685706 & 0.052629 \end{bmatrix} \quad \det[M] = 1$$

$$I_1 \approx 1.83699 \, m \, a^2 \quad I_2 \approx 4.99288 \, m \, a^2 \quad I_3 \approx 6.33679 \, m \, a^2$$

$$\underline{\xi}_1 = 0.642082 \, \underline{N}_1 - 0.246360 \, \underline{N}_2 + 0.725974 \, \underline{N}_3$$

$$\underline{\xi}_2 = -0.648393 \, \underline{N}_1 + 0.330748 \, \underline{N}_2 + 0.685706 \, \underline{N}_3$$

$$\underline{\xi}_3 = -0.409045 \, \underline{N}_1 - 0.910995 \, \underline{N}_2 + 0.052629 \, \underline{N}_3$$

2.

$$[I_o] = m \ell^2 \begin{bmatrix} \frac{1219}{24} & -60 & -60 \\ -60 & \frac{5361}{16} & -20 \\ -60 & -20 & \frac{5361}{16} \end{bmatrix} \approx m \ell^2 \begin{bmatrix} 50.792 & -60 & -60 \\ -60 & 335.06 & -20 \\ -60 & -20 & 335.06 \end{bmatrix}$$

$$[M] \approx \begin{bmatrix} 0.959542 & -0.281564 & 0.000000 \\ 0.199096 & 0.678499 & -0.707107 \\ 0.199096 & 0.678499 & 0.707107 \end{bmatrix} \quad \det[M] = 1$$

$$I_1 \approx 25.8928 \, m \, \ell^2 \quad I_2 \approx 339.961 \, m \, \ell^2 \quad I_3 = 355.062 \, m \, \ell^2$$

$$\underline{\xi}_1 = 0.959542 \, \underline{N}_1 + 0.199096 \, \underline{N}_2 + 0.199096 \, \underline{N}_3$$

$$\underline{\xi}_2 = -0.281564 \, \underline{N}_1 + 0.678499 \, \underline{N}_2 + 0.678499 \, \underline{N}_3$$

$$\underline{\xi}_3 = -0.707107 \, \underline{N}_2 + 0.707107 \, \underline{N}_3$$