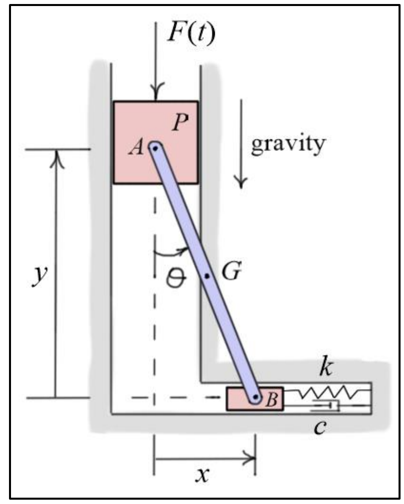
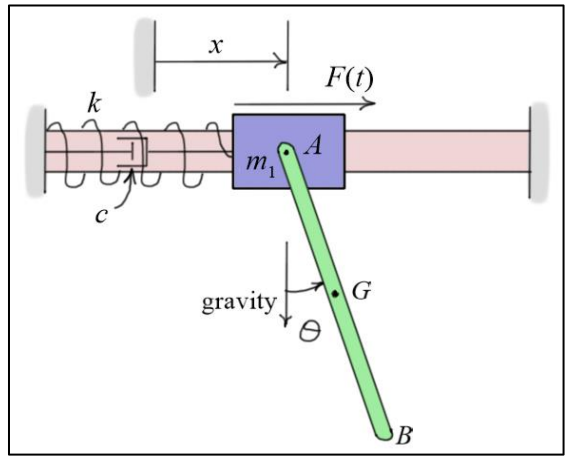
Multibody Dynamics
Exercises #7 – Use d'Alembert's Principle

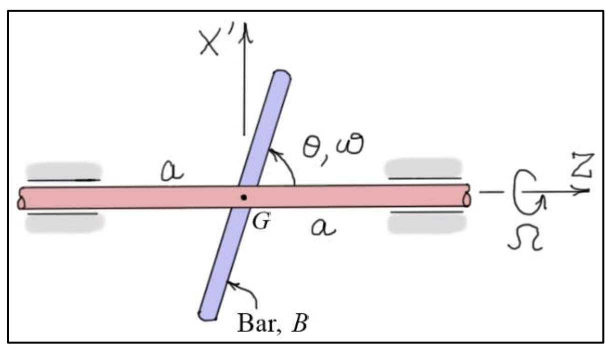
1) Find the differential equation of motion of the single degree-of-freedom system shown. The system consists of slender bar *AB* of mass *m* and length ℓ and a piston *P* of mass m_p. The system is driven by the force $F(t) = F_0 + F_1 \sin(\omega t)$ and gravity. A spring and damper are attached to the light slider at *B*. The spring is unstretched when $x = 0$. Use θ as the generalized coordinate. Neglect friction.



2) Find the differential equations of motion of the two degree-of-freedom system shown. The system consists of a mass m_1 that moves along a fixed horizontal bar and a slender bar *AB* pinned to m_1 at *A*. Bar *AB* has mass m_2 and length ℓ. Mass m_1 is attached to the fixed support by a spring of stiffness *k* and a viscous damper with damping coefficient *c*. The spring is unstretched when $x = 0$. The system is driven by gravity and the force $F(t) = F_0 \sin(\omega t)$ applied to m_1. Use *x* and θ as the generalized coordinates. Neglect friction.



3) Find the differential equations of motion of the two degree-of-freedom system shown. The system consists of a slender bar *B* of length ℓ and mass m pinned through the center of a shaft of mass m_s and radius *r*. The rotation of the shaft about the Z-axis is described by the angle ϕ $(\dot{\phi} = \Omega)$, and the rotation of bar *B* about the Y'-axis axis is described by the angle θ $(\dot{\theta} = \omega)$. A motor torque M_ϕ is applied to the shaft about the Z-axis, and a motor torque M_θ is applied to *B* about the Y'-axis. Use θ and ϕ as the generalized coordinates.



4) Find the differential equations of motion of the two degree-of-freedom system shown. The system consists of a disk *D* of mass m_d and radius *R*, and a slender bar *B* of mass *m* and length ℓ. The rotation of the disk about the Z-axis is described by the angle $\phi\ (\dot{\phi} = \Omega)$, and the rotation of the bar *B* about the X'-axis is described by the angle $\theta\ (\dot{\theta} = \omega)$. A rotational spring-damper is located between *B* and *D* at the pin *P*. The spring has stiffness *k* and is unstretched when $\theta = 0$. The damper has coefficient *c*. A motor torque M_ϕ is applied to the disk about the Z-axis, and a motor torque M_θ is applied to *B* about the X'- axis . Use θ and ϕ as the generalized coordinates.