Multibody Dynamics
Exercises #9 Answers

1. There are ***four*** equations for $\left(\phi, \theta, \omega_1', \omega_2'\right)$:

$$\boxed{\begin{aligned} \left(m\ell^2/12\right)S_\theta\dot{\omega}_1' + \left(m\ell^2/12\right)C_\theta\omega_1'\omega_2' &= -M_\phi \\ \left(m\ell^2/12\right)\dot{\omega}_2' - \left(m\ell^2 C_\theta/12S_\theta\right)\omega_1'^2 &= M_\theta \end{aligned}}$$

with the kinematical differential equations

$$\boxed{\begin{aligned} \dot{\phi} &= -\omega_1'/S_\theta \\ \dot{\theta} &= \omega_2' \end{aligned}}$$

2. There are ***seven*** equations for $\left(\omega_1', \omega_2', \omega_3', \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\right)$:

$$\boxed{\begin{aligned} \left(I + mL^2\right)\dot{\omega}_1' + \left(I_3 - I - mL^2\right)\omega_2'\omega_3' &= 2mgL\left(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4\right) \\ \left(I + mL^2\right)\dot{\omega}_2' + \left(I + mL^2 - I_3\right)\omega_1'\omega_3' &= 2mgL\left(\varepsilon_2\varepsilon_4 - \varepsilon_1\varepsilon_3\right) \\ \omega_3' &= \text{ constant} \end{aligned}}$$

with the kinematical differential equations

$$\boxed{\left\{\dot{\varepsilon}\right\}_{4\times1} = \frac{1}{2}\begin{bmatrix} \varepsilon_4 & -\varepsilon_3 & \varepsilon_2 & \varepsilon_1 \\ \varepsilon_3 & \varepsilon_4 & -\varepsilon_1 & \varepsilon_2 \\ -\varepsilon_2 & \varepsilon_1 & \varepsilon_4 & \varepsilon_3 \\ -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & \varepsilon_4 \end{bmatrix}\begin{Bmatrix} \omega_1' \\ \omega_2' \\ \omega_3' \\ 0 \end{Bmatrix}}$$

3. a) Partial velocities and partial angular velocities

$$\partial \underset{\sim}{v}_G / \partial\omega_1' = L\underset{\sim}{e}_3/2 \qquad \partial \underset{\sim}{v}_G / \partial\omega_2' = 0 \qquad \partial \underset{\sim}{v}_G / \partial\omega_3' = -L\underset{\sim}{e}_1/2$$

$$\partial \underset{\sim}{v}_A / \partial\omega_1' = L\underset{\sim}{e}_3 \qquad \partial \underset{\sim}{v}_A / \partial\omega_2' = 0 \qquad \partial \underset{\sim}{v}_A / \partial\omega_3' = -L\underset{\sim}{e}_1$$

$$\partial \underset{\sim}{v}_B / \partial\omega_1' = L\underset{\sim}{e}_3/2 \qquad \partial \underset{\sim}{v}_B / \partial\omega_2' = -L\underset{\sim}{e}_3/2 \qquad \partial \underset{\sim}{v}_B / \partial\omega_3' = (L/2)(-\underset{\sim}{e}_1 + \underset{\sim}{e}_2)$$

b) Generalized forces

$$\boxed{F_{\omega_1'} = LF_{A3} + LF_B/2 + mgLS_1C_2} \quad \boxed{F_{\omega_2'} = -LF_B/2} \quad \boxed{F_{\omega_3'} = -LF_{A1} + mgL(C_1S_3 + S_1S_2C_3)}$$

c)

$$\boxed{\begin{aligned} \left(\frac{7}{12}mL^2\right)\dot{\omega}_1' + \left(\frac{7}{12}mL^2\right)\omega_2'\omega_3' &= F_{\omega_1'} \\ \left(\frac{1}{12}mL^2\right)\dot{\omega}_2' - \left(\frac{1}{12}mL^2\right)\omega_1'\omega_3' &= F_{\omega_2'} \\ \left(\frac{2}{3}mL^2\right)\dot{\omega}_3' - \left(\frac{1}{2}mL^2\right)\omega_1'\omega_2' &= F_{\omega_3'} \end{aligned}}$$

d)

$$\boxed{\begin{aligned} \dot{\theta}_1 &= \left(\omega_1'C_3 - \omega_2'S_3\right)/C_2 \\ \dot{\theta}_2 &= \omega_1'S_3 + \omega_2'C_3 \\ \dot{\theta}_3 &= \omega_3' + \left(-\omega_1'C_3 + \omega_2'S_3\right)S_2/C_2 \end{aligned}}$$