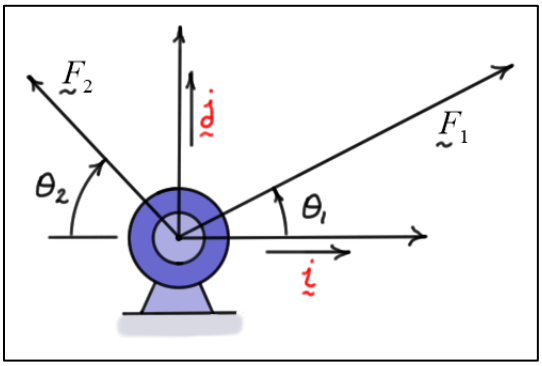
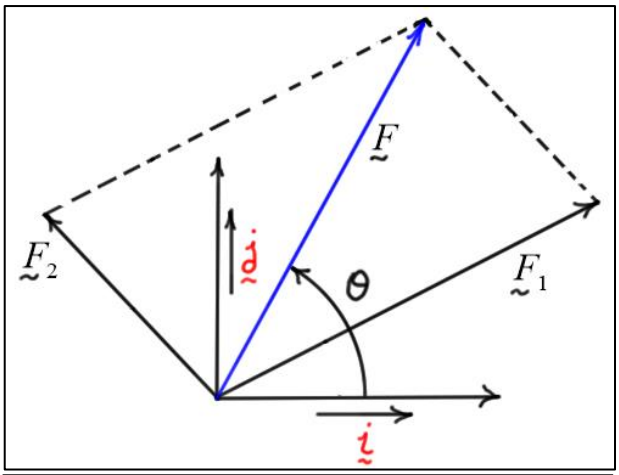
Elementary Statics
Parallelogram Law of Vector Addition

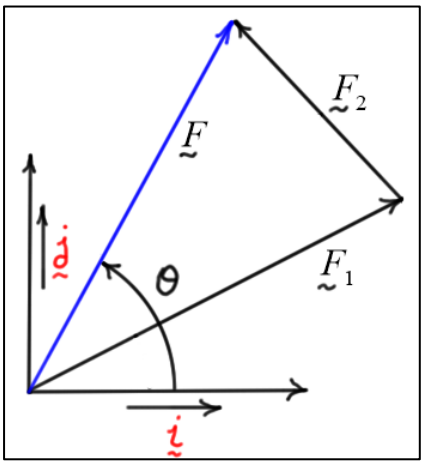
- The most basic way to add two vectors is a ***geometric approach*** called the ***parallelogram law*** of ***vector addition***.
- The ***sum*** of the two vectors is called the ***resultant***.
- Consider the two forces $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ acting on the support as shown in the diagram. The total force acting on the support is found by ***adding*** the two vectors.



- By placing the tails of the vectors at the same point, we can ***construct*** the resultant $\underset{\sim}{F} \triangleq \underset{\sim}{F}_1 + \underset{\sim}{F}_2$ by forming a ***parallelogram*** as shown in the diagram below on the left. By placing the tail of the second vector at the head of the first, we can construct the resultant by forming a ***triangle*** as shown in the diagram on the right.



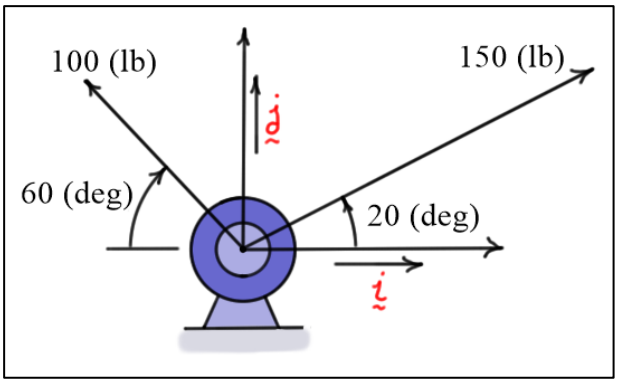
Parallelogram formed by $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$



Triangle formed by $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$

- In general, the triangle formed by $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$, and the resultant $\underset{\sim}{F}$ is a ***non-right triangle***. The lengths and angles within this triangle can be related using the ***law of cosines*** and the ***law of sines***.

Example #1:

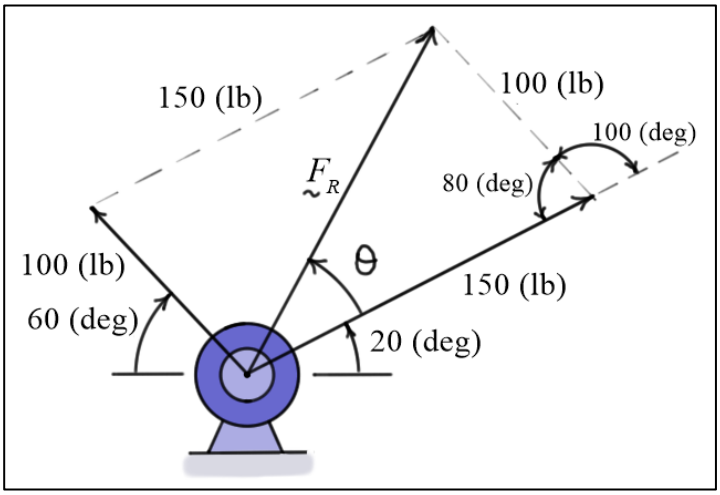


Given: Two forces in the diagram

Find: $\underset{\sim}{F}_R$ the resultant force

Solution:

Geometric construction:



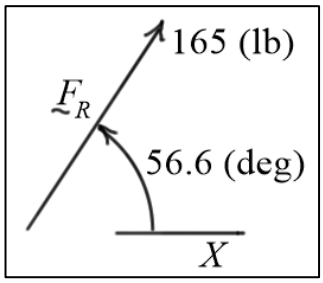
Applying the ***law of cosines*** and ***law of sines*** to the diagram above gives.

Law of Cosines: $F_R^2 = 150^2 + 100^2 - 2(100)(150)\cos(80) \approx 27290.6$

$$\Rightarrow \boxed{F_R \approx 165.199 \approx 165 \text{ (lb)}}$$

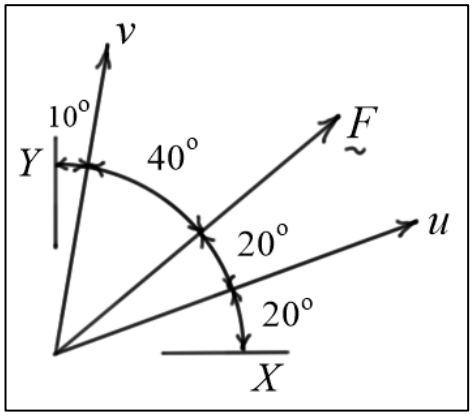
Law of Sines: $\dfrac{\sin(\theta)}{100} = \dfrac{\sin(80)}{F_R} \quad \Rightarrow \boxed{\theta = \sin^{-1}\left(\dfrac{100\sin(80)}{F_R}\right) \approx 36.5937 \approx 36.6 \text{ (deg)}}$

Result: $\underset{\sim}{F}_R$ is a 165 (lb) force acting at 56.6 (deg) relative to the X direction.



Example #2:

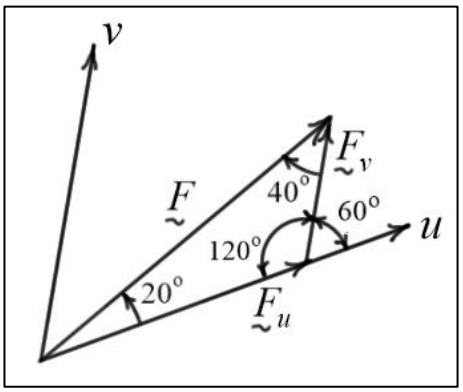
Given: $\left|\underset{\sim}{F}\right| = 200$ (lb) is oriented as shown in the diagram



Find: $\underset{\sim}{F}_u$ and $\underset{\sim}{F}_v$ the components of $\underset{\sim}{F}$ along the *u* and *v* directions

Solution:

Geometric construction:



As drawn, $\underset{\sim}{F} = \underset{\sim}{F}_u + \underset{\sim}{F}_v$. Using the ***law of sines***, write

$$\boxed{\frac{F_u}{\sin(40)} = \frac{F_v}{\sin(20)} = \frac{F}{\sin(120)}}$$

$$\Rightarrow \boxed{F_u = \left(\frac{\sin(40)}{\sin(120)}\right)F = \left(\frac{\sin(40)}{\sin(120)}\right)200 \approx 148.445 \approx 148 \text{ (lb)}}$$

$$\Rightarrow \boxed{F_v = \left(\frac{\sin(20)}{\sin(120)}\right)F = \left(\frac{\sin(20)}{\sin(120)}\right)200 \approx 78.9862 \approx 79.0 \text{ (lb)}}$$