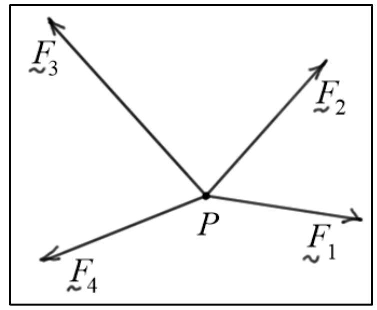
Elementary Statics
Equilibrium of a Particle

- If a particle P does ***not move***, it is said to be in ***static equilibrium***.
- To study the ***forces*** necessary for equilibrium, we can isolate P as a ***free body***, and ***identify*** all the ***forces acting on it***. This is called a ***free body diagram***, and it is necessary to be clear about ***how*** the forces act on P.



Free Body Diagram

- For P to be in ***static equilibrium***, the ***sum*** of all forces (or resultant force) acting on it must be ***zero***. The diagram shows a particle with ***four forces*** acting on it. For a particle with N forces acting on it.

$$\boxed{\sum_{i=1}^{N} \underset{\sim}{F}_i = \underset{\sim}{0}}$$

This means that the ***sum*** of the forces in ***all directions*** must the ***zero***.

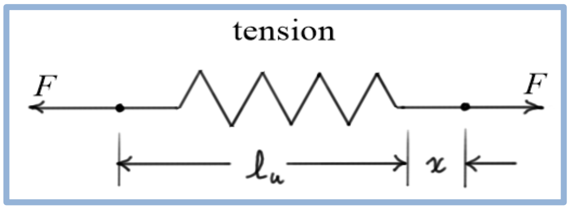
- In two dimensions (2D), this ***vector equation*** represents ***two scalar equations***, and in three dimensions (3D), it represents ***three scalar equations***.

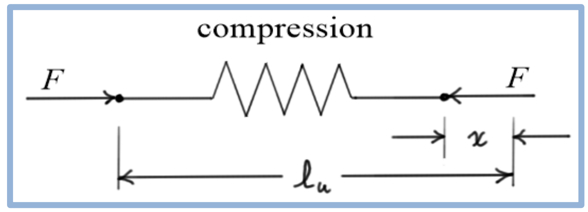
$$\boxed{\begin{aligned}\sum_i (F_x)_i &= 0\\ \sum_i (F_y)_i &= 0\end{aligned}} \quad \text{(2D)} \qquad\qquad \boxed{\begin{aligned}\sum_i (F_x)_i &= 0\\ \sum_i (F_y)_i &= 0\\ \sum_i (F_z)_i &= 0\end{aligned}} \quad \text{(3D)}$$

- The ***free body diagram*** often contains ***unknown forces*** we are trying to find. In 2D problems, we can find up to ***two unknowns***, and in 3D problems, we can find up to ***three unknowns***.

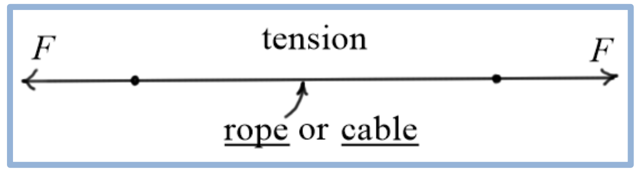
Assumptions for Some Typical Structural Components

- ***Springs*** are assumed to generate forces ***proportional*** to the ***elongation*** or ***compression*** of the spring from its ***natural*** (or ***unstretched***) length, ℓ_u. When stretched, the spring is said to be in "***tension***" and when it is compressed, it is said to be in "***compression***." The force required to hold the spring in tension or compression is $\boxed{F = k\,x}$.

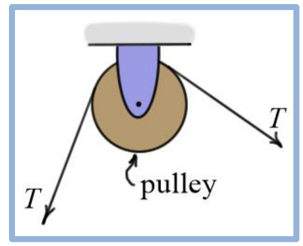




- As a first approximation, ***cables*** or ***ropes*** are assumed to form ***straight lines*** with ***no sag***. They can only act in ***tension***. Forces are transmitted along the line.



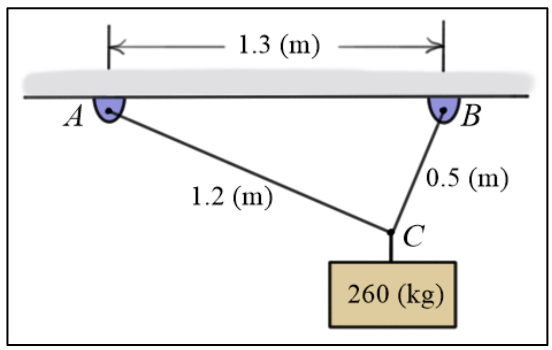
- Forces are often ***redirected*** using ***pulleys***. As a first approximation, assume pulleys are ***massless*** and ***frictionless***, so the tension in the rope or cable is the same on both sides of the pulley.



Example #1:

Given: Two cables support the 260 (kg) load.

Find: Tensions in the two cables.



Solution:

Geometry: Using the law of cosines, write

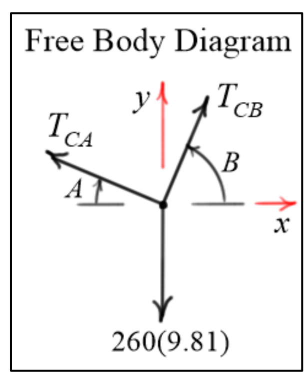
$$0.5^2 = 1.2^2 + 1.3^2 - 2(1.2)(1.3)\cos(A) \quad \Rightarrow \quad \boxed{A = \cos^{-1}\left(\frac{1.2^2 + 1.3^2 - 0.5^2}{2(1.2)(1.3)}\right) \approx 22.6199 \text{ (deg)}}$$

$$1.2^2 = 0.5^2 + 1.3^2 - 2(0.5)(1.3)\cos(B) \quad \Rightarrow \quad \boxed{B = \cos^{-1}\left(\frac{0.5^2 + 1.3^2 - 1.2^2}{2(0.5)(1.3)}\right) \approx 67.3801 \text{ (deg)}}$$

Equations of equilibrium:

$$\boxed{\sum F_x = -\cos(A)T_{CA} + \cos(B)T_{CB} = 0}$$

$$\boxed{\sum F_y = \sin(A)T_{CA} + \sin(B)T_{CB} - 260(9.81) = 0}$$



Simultaneous equations:

$$\boxed{\begin{bmatrix} -\cos(A) & \cos(B) \\ \sin(A) & \sin(B) \end{bmatrix} \begin{Bmatrix} T_{CA} \\ T_{CB} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 260(9.81) \end{Bmatrix}} \quad \Rightarrow \quad \boxed{\begin{Bmatrix} T_{CA} \\ T_{CB} \end{Bmatrix} \approx \begin{Bmatrix} 981 \text{ (N)} \\ 2350 \text{ (N)} \end{Bmatrix}}$$

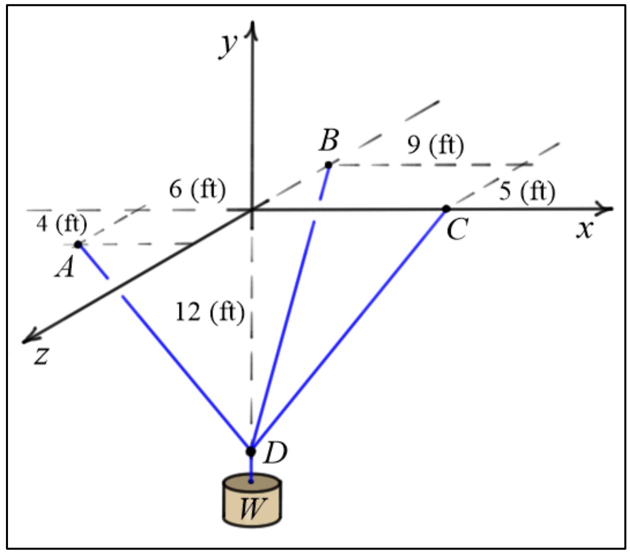
Example #2:

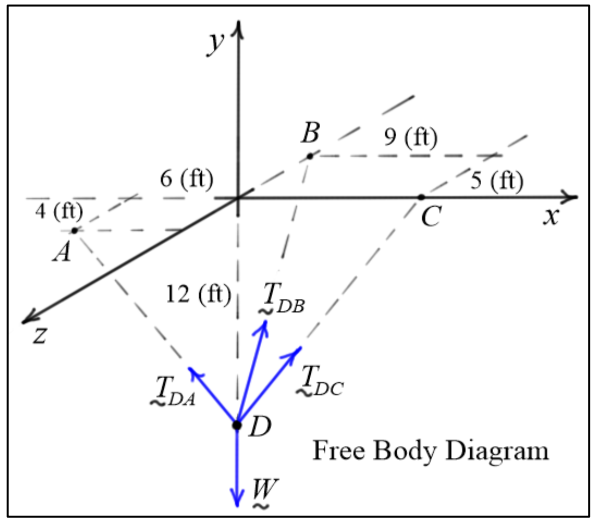
Given:

Three cables support weight W as shown. Tension in cable DB is measured to be 975 (lb).

Find:

Tensions in cables DA and DC and weight W.





Solution:

$$\boxed{\underset{\sim}{W} = -W\,\underset{\sim}{j}}$$

$$\underset{\sim}{T}_{DB} = 975\,\underset{\sim}{n}_{DB} = 975\left(12\,\underset{\sim}{j} - 5\,\underset{\sim}{k}\right)\Big/\sqrt{12^2+5^2}$$

$$= 975\left(12\,\underset{\sim}{j} - 5\,\underset{\sim}{k}\right)\Big/13 \quad \Rightarrow \boxed{\underset{\sim}{T}_{DB} = 900\,\underset{\sim}{j} - 375\,\underset{\sim}{k}}$$

$$\underset{\sim}{T}_{DC} = T_{DC}\,\underset{\sim}{n}_{DC} = T_{DC}\left(9\,\underset{\sim}{i} + 12\,\underset{\sim}{j}\right)\Big/\sqrt{9^2+12^2}$$

$$= T_{DC}\left(9\,\underset{\sim}{i} + 12\,\underset{\sim}{j}\right)\Big/15 \quad \Rightarrow \boxed{\underset{\sim}{T}_{DC} = T_{DC}\left(3\,\underset{\sim}{i} + 4\,\underset{\sim}{j}\right)\Big/5}$$

$$\underset{\sim}{T}_{DA} = T_{DA}\,\underset{\sim}{n}_{DA} = T_{DA}\left(-6\,\underset{\sim}{i} + 12\,\underset{\sim}{j} + 4\,\underset{\sim}{k}\right)\Big/\sqrt{6^2+12^2+4^2}$$

$$= T_{DA}\left(-6\,\underset{\sim}{i} + 12\,\underset{\sim}{j} + 4\,\underset{\sim}{k}\right)\Big/14$$

$$\Rightarrow \boxed{\underset{\sim}{T}_{DA} = T_{DA}\left(-3\,\underset{\sim}{i} + 6\,\underset{\sim}{j} + 2\,\underset{\sim}{k}\right)\Big/7}$$

Equations of Equilibrium:

$$\boxed{\sum F_x = -\tfrac{3}{7}T_{DA} + \tfrac{3}{5}T_{DC} = 0} \quad \boxed{\sum F_y = \tfrac{6}{7}T_{DA} + \tfrac{4}{5}T_{DC} + 900 - W = 0} \quad \boxed{\sum F_z = \tfrac{2}{7}T_{DA} - 375 = 0}$$

Solving:

The third equation gives

$$\boxed{T_{DA} = \tfrac{7}{2}(375) = 1312.5 \approx 1310 \text{ (lb)}}$$

Substituting this result into the first equation gives

$$\boxed{T_{DC} = \left(\tfrac{5}{3}\right)\left(\tfrac{3}{7}\right)T_{DA} = 937.5 \approx 938 \text{ (lb)}}$$

Substituting these two results into the second equation gives

$$\boxed{W = \tfrac{6}{7}T_{DA} + \tfrac{4}{5}T_{DC} + 900 = 2775 \approx 2780 \text{ (lb)}}$$