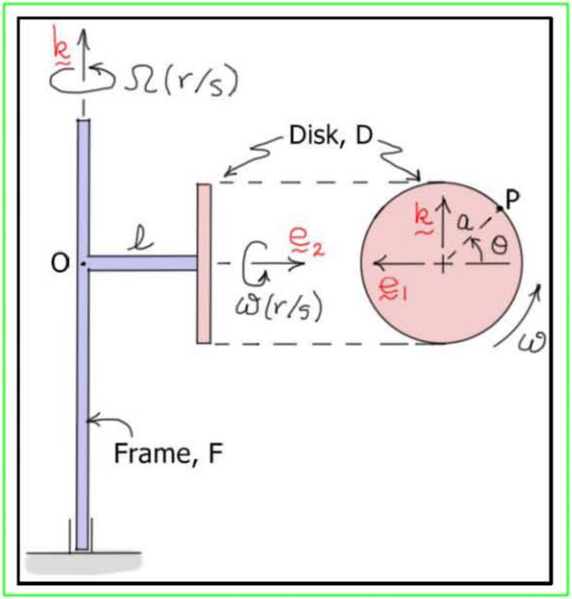
Intermediate Dynamics – Example #1: Angular Velocity and Angular Acceleration

Reference frames:

$R: \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$F: \underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotating frame)



Find:

a) ${}^R\underset{\sim}{\omega}_D$ … the ***angular velocity*** of disk D in R

b) ${}^R\underset{\sim}{\alpha}_D$ … the ***angular acceleration*** of disk D in R

Solution: summation rule

a) $${}^R\underset{\sim}{\omega}_D = {}^F\underset{\sim}{\omega}_D + {}^R\underset{\sim}{\omega}_F = \omega\, \underset{\sim}{e}_2 + \Omega\, \underset{\sim}{k} \quad \text{(r/s)}$$

b) $${}^R\underset{\sim}{\alpha}_D = \frac{{}^R d}{dt}\left({}^R\underset{\sim}{\omega}_D\right) = \dot{\omega}\, \underset{\sim}{e}_2 + \omega \frac{{}^R d}{dt}\left(\underset{\sim}{e}_2\right) + \dot{\Omega}\, \underset{\sim}{k} + \Omega \underbrace{\frac{{}^R d}{dt}\left(\underset{\sim}{k}\right)}_{\text{zero}}$$ (using direct differentiation)

$$= \dot{\omega}\, \underset{\sim}{e}_2 + \omega\left({}^R\underset{\sim}{\omega}_F \times \underset{\sim}{e}_2\right) + \dot{\Omega}\, \underset{\sim}{k} = \dot{\omega}\, \underset{\sim}{e}_2 + \omega\left(\Omega\, \underset{\sim}{k} \times \underset{\sim}{e}_2\right) + \dot{\Omega}\, \underset{\sim}{k}$$

So,

$${}^R\underset{\sim}{\alpha}_D = -\omega\,\Omega\, \underset{\sim}{e}_1 + \dot{\omega}\, \underset{\sim}{e}_2 + \dot{\Omega}\, \underset{\sim}{k} \quad (\text{r/s}^2)$$