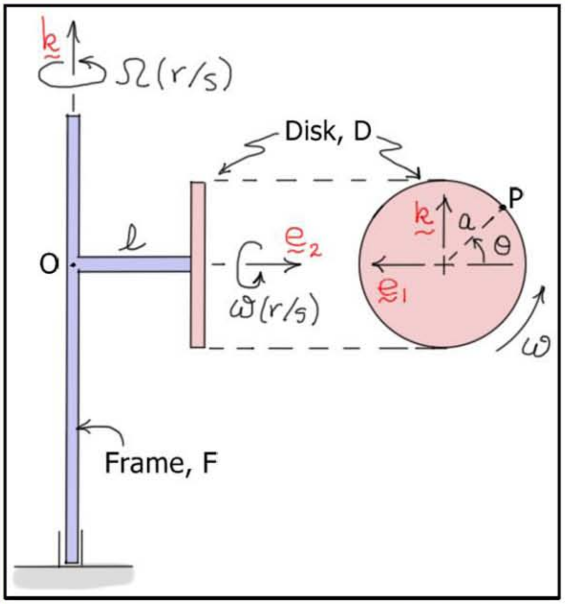
<u>Example #3 – Intermediate Dynamics: Acceleration</u>



Reference frames:

R : $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

F : $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (fixed in the rotating frame)

Find:

${}^R\underset{\sim}{a}_P$ … the ***acceleration*** of point P in R using ***direct differentiation***

Solution:

To find the acceleration of P, we can differentiate the velocity vector of P. In this solution, we take advantage again of the "derivative rule."

$$ {}^R\underset{\sim}{a}_P = \frac{{}^Rd}{dt}\left({}^R\underset{\sim}{v}_P\right) = \frac{{}^Fd}{dt}\left({}^R\underset{\sim}{v}_P\right) + {}^R\underset{\sim}{\omega}_F \times \left({}^R\underset{\sim}{v}_P\right) $$

(previous result)

Here,

$$ \frac{{}^Fd}{dt}\left({}^R\underset{\sim}{v}_P\right) = \frac{{}^Fd}{dt}\Big(\left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 - \left(a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}\Big) $$

$$ = \left(a\dot{\omega} S_\theta + a\omega^2 C_\theta - \ell\dot{\Omega}\right)\underset{\sim}{e}_1 - \left(a\dot{\Omega}C_\theta - a\Omega\omega S_\theta\right)\underset{\sim}{e}_2 + \left(a\dot{\omega}C_\theta - a\omega^2 S_\theta\right)\underset{\sim}{k} $$

$$ {}^R\underset{\sim}{\omega}_F \times {}^R\underset{\sim}{v}_P = \left(\Omega\,\underset{\sim}{k}\right)\times\left[\left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 - \left(a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}\right] $$

$$ = \left(a\Omega^2 C_\theta\right)\underset{\sim}{e}_1 + \left(\left(a\omega S_\theta - \ell\Omega\right)\Omega\right)\underset{\sim}{e}_2 $$

Aside:

$$ \frac{d}{dt}\left(a\omega S_\theta\right) = a\dot{\omega} S_\theta + a\omega\left(\dot{\theta} C_\theta\right) = a\dot{\omega} S_\theta + a\omega^2 C_\theta $$

$$ \frac{d}{dt}\left(a\Omega S_\theta\right) = a\dot{\Omega} S_\theta + a\Omega\omega C_\theta $$

Substituting these results into the boxed equation gives the result.

$$ {}^R\underset{\sim}{a}_P = \left[a\dot{\omega} S_\theta - \ell\dot{\Omega} + aC_\theta\left(\omega^2 + \Omega^2\right)\right]\underset{\sim}{e}_1 + \left[-a\dot{\Omega}C_\theta + 2a\omega\Omega S_\theta - \ell\Omega^2\right]\underset{\sim}{e}_2 + \left[a\dot{\omega}C_\theta - a\omega^2 S_\theta\right]\underset{\sim}{k} $$