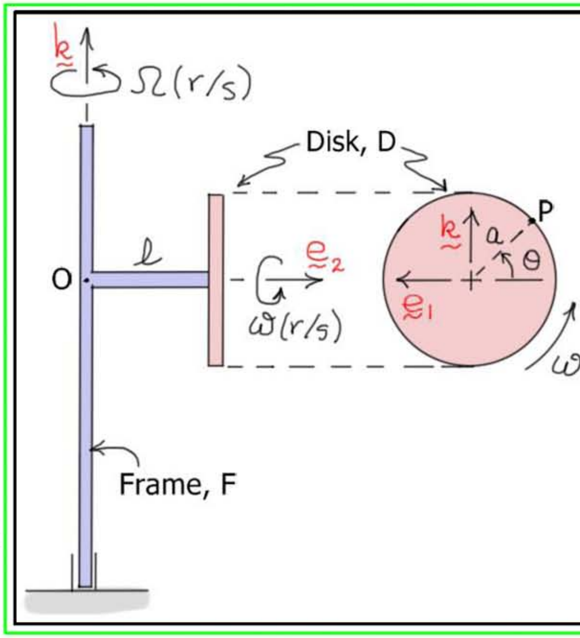
Example #4 – Intermediate Dynamics: Velocity – Two-Point Formula



Reference frames:

$R: \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$F: \underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotating frame)

Find:

${}^R\underset{\sim}{v}_P$ ... the ***velocity*** of point P in R using the ***two-point formula***

Solution:

To find the velocity of P, we can use the formula that relates the velocity of two points on a rigid body.

Result from a previous example: ${}^R\underset{\sim}{\omega}_D = \omega\, \underset{\sim}{e}_2 + \Omega\, \underset{\sim}{k}$

Using the formula that relates the velocity of two points of a rigid body, write

$${}^R\underset{\sim}{v}_P = {}^R\underset{\sim}{v}_Q + {}^R\underset{\sim}{v}_{P/Q}$$ (here, Q represents the center of the disk)

where,

$${}^R\underset{\sim}{v}_Q = -\ell\,\Omega\,\underset{\sim}{e}_1$$ (Q has circular motion around O)

$${}^R\underset{\sim}{v}_{P/Q} = {}^R\underset{\sim}{\omega}_D \times \underset{\sim}{r}_{P/Q} = \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{k} \\ 0 & \omega & \Omega \\ -aC_\theta & 0 & aS_\theta \end{vmatrix}$$

$$= \left(a\omega S_\theta\right)\underset{\sim}{e}_1 + \left(-a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}$$

Adding these two results gives

$${}^R\underset{\sim}{v}_P = \left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 - \left(a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}$$