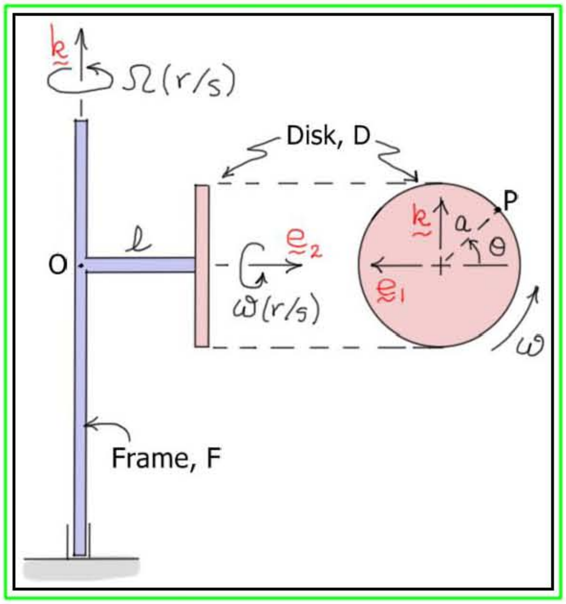
# Example #5 – Intermediate Dynamics: Acceleration – Two-Point Formula




Reference frames:

$R: \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$F: \underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotating frame)

Find:

${}^R\underset{\sim}{a}_P$ … the ***acceleration*** of point $P$ in $R$ using the ***two-point formula***

Solution:

To find the acceleration of $P$, use the formula that relates the accelerations of two points of a rigid body.

$${}^R\underset{\sim}{\omega}_D = \omega\,\underset{\sim}{e}_2 + \Omega\,\underset{\sim}{k} \qquad {}^R\underset{\sim}{\alpha}_D = -\omega\,\Omega\,\underset{\sim}{e}_1 + \dot{\omega}\,\underset{\sim}{e}_2 + \dot{\Omega}\,\underset{\sim}{k}$$

$${}^R\underset{\sim}{v}_{P/Q} = \left(a\omega S_\theta\right)\underset{\sim}{e}_1 + \left(-a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}$$

Using the two-point formula, write:

$${}^R\underset{\sim}{a}_P = {}^R\underset{\sim}{a}_Q + \left({}^R\underset{\sim}{\alpha}_D \times \underset{\sim}{r}_{P/Q}\right) + \left({}^R\underset{\sim}{\omega}_D \times {}^R\underset{\sim}{v}_{P/Q}\right)$$

(for two points fixed on a rigid body)

Here,

$${}^R\underset{\sim}{a}_Q = -\ell\dot{\Omega}\,\underset{\sim}{e}_1 - \ell\Omega^2\,\underset{\sim}{e}_2$$ ($Q$ has circular motion around $O$)

$${}^R\underset{\sim}{\alpha}_D \times \underset{\sim}{r}_{P/Q} = \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{k} \\ -\omega\Omega & \dot{\omega} & \dot{\Omega} \\ -aC_\theta & 0 & aS_\theta \end{vmatrix} = \left(a\dot{\omega}S_\theta\right)\underset{\sim}{e}_1 + \left(a\omega\Omega S_\theta - a\dot{\Omega}C_\theta\right)\underset{\sim}{e}_2 + \left(a\dot{\omega}C_\theta\right)\underset{\sim}{k}$$

$${}^R\underset{\sim}{\omega}_D \times {}^R\underset{\sim}{v}_{P/Q} = \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{k} \\ 0 & \omega & \Omega \\ a\omega S_\theta & -a\Omega C_\theta & a\omega C_\theta \end{vmatrix}$$

$$= \left(a\omega^2 C_\theta + a\Omega^2 C_\theta\right)\underset{\sim}{e}_1 + \left(a\omega\Omega S_\theta\right)\underset{\sim}{e}_2 + \left(-a\omega^2 S_\theta\right)\underset{\sim}{k}$$

Substituting these results into the boxed equation gives

$${}^R\underset{\sim}{a}_P = \left[a\dot{\omega}S_\theta - \ell\dot{\Omega} + aC_\theta\left(\omega^2+\Omega^2\right)\right]\underset{\sim}{e}_1 + \left[-a\dot{\Omega}C_\theta + 2a\omega\Omega S_\theta - \ell\Omega^2\right]\underset{\sim}{e}_2 + \left[a\dot{\omega}C_\theta - a\omega^2 S_\theta\right]\underset{\sim}{k}$$