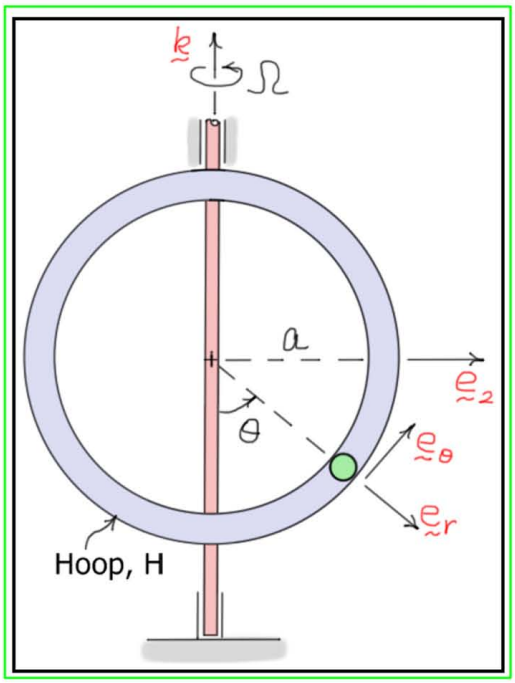
Example #6 – Intermediate Dynamics: Point Moving on a Rigid Body

Reference frames:

R: $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

H: $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotates with the hoop)

P: $\underset{\sim}{e}_1, \underset{\sim}{e}_r, \underset{\sim}{e}_\theta$ (rotates with the hoop and ball)



Find:

${}^R\underset{\sim}{v}_P$ … the ***velocity*** of point P in R

${}^R\underset{\sim}{a}_P$ … the ***acceleration*** of point P in R

Solution:

As ***hoop*** H rotates about the ***vertical axis***, the ball P slides within the hoop. To find the velocity and acceleration of P, use the formulae for ***points moving on bodies***. (for a point moving on a body)

Velocity: $\boxed{{}^R\underset{\sim}{v}_P = {}^R\underset{\sim}{v}_{\hat{P}} + {}^H\underset{\sim}{v}_P}$ (here $\hat{P}$ is a point fixed on H and coinciding with P)

Here,

$\boxed{{}^R\underset{\sim}{v}_{\hat{P}} = -aS_\theta \Omega\, \underset{\sim}{e}_1}$ ($\hat{P}$ has circular motion around the vertical axis)

$\boxed{{}^H\underset{\sim}{v}_P = a\dot{\theta}\, \underset{\sim}{e}_\theta = a\dot{\theta}\left(C_\theta\, \underset{\sim}{e}_2 + S_\theta\, \underset{\sim}{k}\right)}$ (circular motion of P on H)

Adding these two results, gives the velocity of P:

$$\boxed{{}^R\underset{\sim}{v}_P = -aS_\theta \Omega\, \underset{\sim}{e}_1 + a\dot{\theta}C_\theta\, \underset{\sim}{e}_2 + a\dot{\theta}S_\theta\, \underset{\sim}{k}}$$

Acceleration: $\boxed{{}^R\underset{\sim}{a}_P = {}^R\underset{\sim}{a}_{\hat{P}} + {}^H\underset{\sim}{a}_P + 2\left({}^R\underset{\sim}{\omega}_H \times {}^H\underset{\sim}{v}_P\right)}$ (for a point moving on a body)

Here,

$\boxed{{}^R\underset{\sim}{a}_{\hat{P}} = -aS_\theta \dot{\Omega}\, \underset{\sim}{e}_1 - aS_\theta \Omega^2\, \underset{\sim}{e}_2}$ (circular motion of $\hat{P}$)

$\boxed{{}^H\underset{\sim}{a}_P = a\ddot{\theta}\, \underset{\sim}{e}_\theta - a\dot{\theta}^2\, \underset{\sim}{e}_r}$ (circular motion of P on H)

$\boxed{2\left({}^R\underset{\sim}{\omega}_H \times {}^H\underset{\sim}{v}_P\right) = 2\Omega\, \underset{\sim}{k} \times a\dot{\theta}\left(C_\theta\, \underset{\sim}{e}_2 + S_\theta\, \underset{\sim}{k}\right) = -2a\dot{\theta}\Omega C_\theta\, \underset{\sim}{e}_1}$ (Coriolis acceleration)

Adding these two results, gives the acceleration of P:

$$\boxed{{}^R\underset{\sim}{a}_P = -\left(a\dot{\Omega}S_\theta + 2a\dot{\theta}\Omega C_\theta\right)\underset{\sim}{e}_1 + \left(a\ddot{\theta}C_\theta - a\dot{\theta}^2 S_\theta - a\Omega^2 S_\theta\right)\underset{\sim}{e}_2 + \left(a\ddot{\theta}S_\theta + a\dot{\theta}^2 C_\theta\right)\underset{\sim}{k}}$$