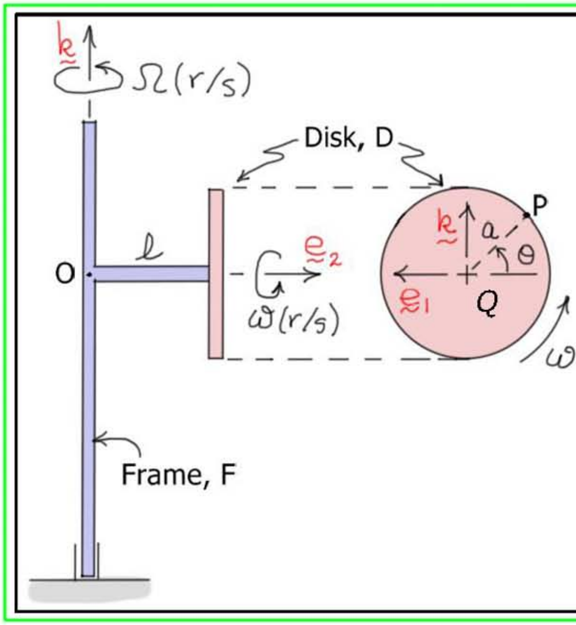
Example #7 – Intermediate Dynamics: Points Moving on Bodies



Reference frames:

$R: \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$F: \underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotating frame)

Find:

${}^R\underset{\sim}{v}_P$ … the ***velocity*** of point P in R using the ***formula for points moving on bodies***

Solution:

To find the velocity of P, use the formula for points moving on a rigid body. In this example, think of point P as moving on the frame F (rather than as a point fixed on the disk).

$${}^R\underset{\sim}{v}_P = {}^R\underset{\sim}{v}_{\hat{P}} + {}^F\underset{\sim}{v}_P$$ (here, $\hat{P}$ is fixed on F and coincides with P)

Here,

$${}^R\underset{\sim}{v}_{\hat{P}} = \underbrace{{}^R\underset{\sim}{v}_O}_{\text{zero}} + {}^R\underset{\sim}{v}_{\hat{P}/O} = {}^R\underset{\sim}{\omega}_F \times \underset{\sim}{r}_{\hat{P}/O} = \Omega\,\underset{\sim}{k} \times \left(-aC_\theta\,\underset{\sim}{e}_1 + \ell\,\underset{\sim}{e}_2 + aS_\theta\,\underset{\sim}{k}\right)$$

$$\boxed{{}^R\underset{\sim}{v}_{\hat{P}} = \Omega\left(-aC_\theta\,\underset{\sim}{e}_2 - \ell\,\underset{\sim}{e}_1\right)}$$

$${}^F\underset{\sim}{v}_P = \underbrace{{}^F\underset{\sim}{v}_Q}_{\text{zero}} + {}^F\underset{\sim}{v}_{P/Q} = {}^F\underset{\sim}{\omega}_D \times \underset{\sim}{r}_{P/Q} = \omega\,\underset{\sim}{e}_2 \times \left(-aC_\theta\,\underset{\sim}{e}_1 + aS_\theta\,\underset{\sim}{k}\right)$$

$$\boxed{{}^F\underset{\sim}{v}_P = \omega\left(aC_\theta\,\underset{\sim}{k} + aS_\theta\,\underset{\sim}{e}_1\right)}$$

Adding these two results gives

$$\boxed{{}^R\underset{\sim}{v}_P = \left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 + \left(-a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}}$$