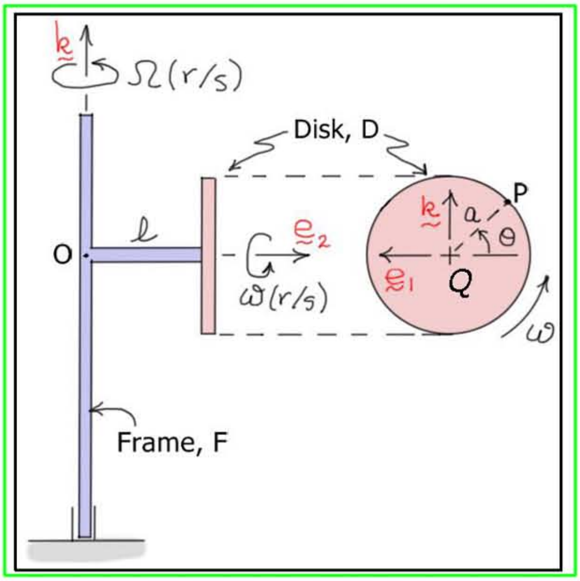
Example #8 – Intermediate Dynamics: Point Moving on a Body

Reference frames:

R : $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

F : $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (rotating frame)



Find:

${}^R\underset{\sim}{a}_P$ … the ***acceleration*** of point P in R using the ***formula for points moving on bodies***

Solution:

To find the ***acceleration*** of P, use the formula for ***a point moving on a rigid body***. In this example, think of point P as ***moving on the frame F*** (rather than as a point fixed on the disk).

$$\boxed{{}^R\underset{\sim}{a}_P = {}^R\underset{\sim}{a}_{\hat{P}} + {}^F\underset{\sim}{a}_P + 2\left({}^R\underset{\sim}{\omega}_F \times {}^F\underset{\sim}{v}_P\right)}$$ (here, $\hat{P}$ is fixed on F and coincides with P)

where,

$${}^R\underset{\sim}{a}_{\hat{P}} = \underbrace{{}^R\underset{\sim}{a}_O}_{\text{zero}} + {}^R\underset{\sim}{a}_{\hat{P}/O} = \left[{}^R\underset{\sim}{\alpha}_F \times \underset{\sim}{r}_{\hat{P}/O}\right] + \left[{}^R\underset{\sim}{\omega}_F \times {}^R\underset{\sim}{v}_{\hat{P}/O}\right]$$ (P_hat and O are both fixed on F)

$$= \left[\dot{\Omega}\,\underset{\sim}{k} \times \left(-aC_\theta\,\underset{\sim}{e}_1 + \ell\,\underset{\sim}{e}_2 + aS_\theta\,\underset{\sim}{k}\right)\right] + \left[\Omega\,\underset{\sim}{k} \times \underbrace{\Omega\left(-aC_\theta\,\underset{\sim}{e}_2 - \ell\,\underset{\sim}{e}_1\right)}_{\text{from previous example}}\right]$$

$$= \dot{\Omega}\left(-aC_\theta\,\underset{\sim}{e}_2 - \ell\,\underset{\sim}{e}_1\right) + \Omega^2\left(aC_\theta\,\underset{\sim}{e}_1 - \ell\,\underset{\sim}{e}_2\right)$$

$$\boxed{{}^R\underset{\sim}{a}_{\hat{P}} = \left(a\Omega^2 C_\theta - \ell\dot{\Omega}\right)\underset{\sim}{e}_1 - \left(\ell\Omega^2 + a\dot{\Omega}C_\theta\right)\underset{\sim}{e}_2}$$

$${}^F\underset{\sim}{a}_P = \left[{}^F\underset{\sim}{\alpha}_D \times \underset{\sim}{r}_{P/Q}\right] + \left[{}^F\underset{\sim}{\omega}_D \times {}^F\underset{\sim}{v}_{P/Q}\right]$$ (P and Q are fixed on the disk)

$$= \left[\dot{\omega}\,\underset{\sim}{e}_2 \times \left(-aC_\theta\,\underset{\sim}{e}_1 + aS_\theta\,\underset{\sim}{k}\right)\right] + \left[\omega\,\underset{\sim}{e}_2 \times \left(\omega\left(aC_\theta\,\underset{\sim}{k} + aS_\theta\,\underset{\sim}{e}_1\right)\right)\right]$$

$$= \dot{\omega}\left(aC_\theta\,\underset{\sim}{k} + aS_\theta\,\underset{\sim}{e}_1\right) + \omega^2\left(aC_\theta\,\underset{\sim}{e}_1 - aS_\theta\,\underset{\sim}{k}\right)$$

$$\boxed{{}^F\underset{\sim}{a}_P = \left(a\dot{\omega}S_\theta + a\omega^2 C_\theta\right)\underset{\sim}{e}_1 + \left(a\dot{\omega}C_\theta - a\omega^2 S_\theta\right)\underset{\sim}{k}}$$

$$\boxed{2\left({}^R\underset{\sim}{\omega}_F \times {}^F\underset{\sim}{v}_P\right) = 2\Omega\,\underset{\sim}{k} \times \omega\left(aC_\theta\,\underset{\sim}{k} + aS_\theta\,\underset{\sim}{e}_1\right) = 2a\omega\Omega S_\theta\,\underset{\sim}{e}_2}$$

Adding these three results gives

$$\boxed{\begin{aligned}{}^R\underset{\sim}{a}_P = {} & \left(a\dot{\omega}S_\theta - \ell\dot{\Omega} + aC_\theta\left(\omega^2 + \Omega^2\right)\right)\underset{\sim}{e}_1 + \left(2a\omega\Omega S_\theta - a\dot{\Omega}C_\theta - \ell\Omega^2\right)\underset{\sim}{e}_2 \\ & + \left(a\dot{\omega}C_\theta - a\omega^2 S_\theta\right)\underset{\sim}{k}\end{aligned}}$$