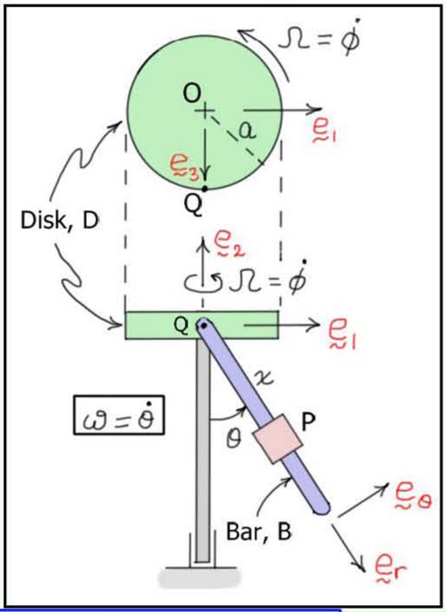
Example #10 – Intermediate Dynamics: Point Moving on a Body

Reference frames: (R is a fixed frame)

D: $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3$ (rotating with disk D)

B: $\underset{\sim}{e}_r, \underset{\sim}{e}_\theta, \underset{\sim}{e}_3$ (rotating with the bar B)



Find:

${}^R\underset{\sim}{a}_P$ ... the ***acceleration*** of point P in R.

Solution:

To find the ***acceleration*** of P, use the formula for a point moving on a rigid body. First need to calculate the angular acceleration of the bar.

Angular Acceleration of Bar, B

$$ {}^R\underset{\sim}{\alpha}_B = \frac{{}^R d\left({}^R\underset{\sim}{\omega}_B\right)}{dt} = \frac{{}^R d}{dt}\left(\Omega\,\underset{\sim}{e}_2 + \omega\,\underset{\sim}{e}_3\right) = \dot{\Omega}\,\underset{\sim}{e}_2 + \dot{\omega}\,\underset{\sim}{e}_3 + \omega\left(\Omega\,\underset{\sim}{e}_2 \times \underset{\sim}{e}_3\right) = \omega\Omega\,\underset{\sim}{e}_1 + \dot{\Omega}\,\underset{\sim}{e}_2 + \dot{\omega}\,\underset{\sim}{e}_3 $$

Now write

$$ {}^R\underset{\sim}{a}_P = {}^R\underset{\sim}{a}_{\hat{P}} + {}^B\underset{\sim}{a}_P + 2\left({}^R\underset{\sim}{\omega}_B \times {}^B\underset{\sim}{v}_P\right) $$ ($\hat{P}$ is fixed on B and coincides with P)

Here,

$$ {}^R\underset{\sim}{a}_{\hat{P}} = {}^R\underset{\sim}{a}_Q + {}^R\underset{\sim}{a}_{\hat{P}/Q} = \left(a\dot{\Omega}\,\underset{\sim}{e}_1 - a\Omega^2\,\underset{\sim}{e}_3\right) + \left({}^R\underset{\sim}{\alpha}_B \times \underset{\sim}{r}_{\hat{P}/Q}\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{v}_{\hat{P}/Q}\right) $$ (P_hat and Q are fixed on B)

$$ = \left(a\dot{\Omega}\,\underset{\sim}{e}_1 - a\Omega^2\,\underset{\sim}{e}_3\right) + \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ \omega\Omega & \dot{\Omega} & \dot{\omega} \\ xS_\theta & -xC_\theta & 0 \end{vmatrix} + \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ x\omega C_\theta & x\omega S_\theta & -x\Omega S_\theta \end{vmatrix} $$

$$ {}^R\underset{\sim}{a}_{\hat{P}} = \left(a\dot{\Omega} + x\dot{\omega}C_\theta - xS_\theta\left(\Omega^2+\omega^2\right)\right)\underset{\sim}{e}_1 + \left(x\dot{\omega}S_\theta + x\omega^2 C_\theta\right)\underset{\sim}{e}_2 - \left(a\Omega^2 + 2x\omega\Omega C_\theta + x\dot{\Omega}S_\theta\right)\underset{\sim}{e}_3 $$

$$ {}^B\underset{\sim}{a}_P = \underbrace{{}^B\underset{\sim}{a}_Q}_{\text{zero}} + {}^B\underset{\sim}{a}_{P/Q} = \ddot{x}\,\underset{\sim}{e}_r = \ddot{x}\left(S_\theta\,\underset{\sim}{e}_1 - C_\theta\,\underset{\sim}{e}_2\right) $$

$$ 2\left({}^R\underset{\sim}{\omega}_B \times {}^B\underset{\sim}{v}_P\right) = 2\begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ \dot{x}S_\theta & -\dot{x}C_\theta & 0 \end{vmatrix} = 2\left[\left(\dot{x}\omega C_\theta\right)\underset{\sim}{e}_1 + \left(\dot{x}\omega S_\theta\right)\underset{\sim}{e}_2 - \left(\dot{x}\Omega S_\theta\right)\underset{\sim}{e}_3\right] $$

Adding these three results gives

$$ \begin{aligned} {}^R\underset{\sim}{a}_P = {} & \left(a\dot{\Omega} + x\dot{\omega}C_\theta - xS_\theta(\Omega^2+\omega^2) + \ddot{x}S_\theta + 2\dot{x}\omega C_\theta\right)\underset{\sim}{e}_1 \\ & + \left(x\dot{\omega}S_\theta + x\omega^2 C_\theta - \ddot{x}C_\theta + 2\dot{x}\omega S_\theta\right)\underset{\sim}{e}_2 - \left(a\Omega^2 + 2x\omega\Omega C_\theta + x\dot{\Omega}S_\theta + 2\dot{x}\Omega S_\theta\right)\underset{\sim}{e}_3 \end{aligned} $$