

Example #14 – Intermediate Dynamics: Simple Crank Shaft – Bearing Loads

The figure shows a **simple crank shaft** consisting of **seven segments**, each considered to be a **slender bar**. Each segment of **length** ℓ has **mass** m . There are six segments of length ℓ and one segment of length 2ℓ (segment 4). The mass center of the system is G and is located on the axis of rotation.

Reference frames:

$$R: \underline{i}, \underline{j}, \underline{k} \text{ (fixed frame)}$$

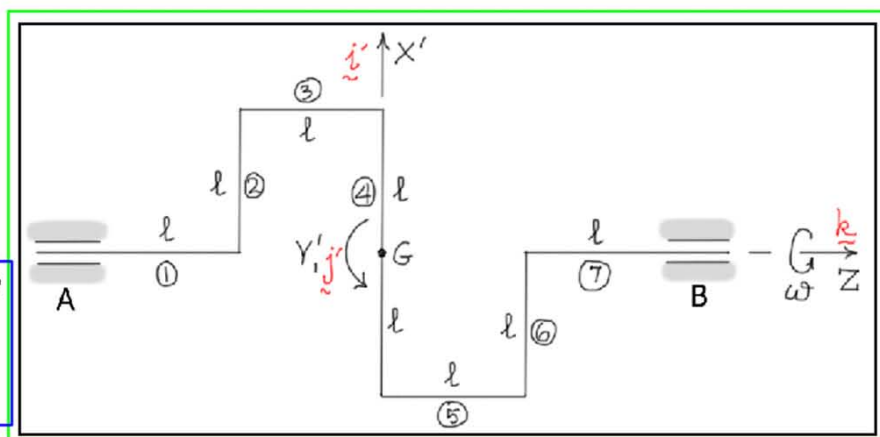
$$S: \underline{i}', \underline{j}', \underline{k} \text{ (rotates with the shaft)}$$

Find:

$\underline{A}, \underline{B}$... the bearing loads at A and B

T ... the driving torque

Neglect weight forces.

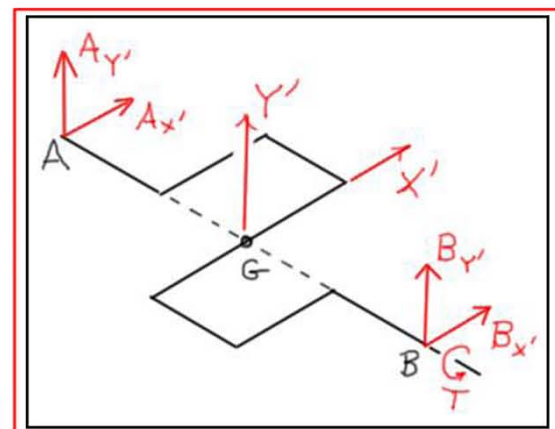


Solution:

Previous results for the **angular momentum**: $\underline{H}_G = 2m\ell^2\omega \underline{i}' + (\frac{10}{3})m\ell^2\omega \underline{k}$

The **bearing loads** at A and B can be found by applying the **Newton/Euler equations of motion** to the **free-body diagram** shown at the right.

$$\begin{aligned} \sum \underline{F} &= m^R \underline{a}_G = \underline{0} \\ \sum \underline{M}_A &= (\underline{I}_{\underline{z}_G} \cdot {}^R \underline{\alpha}_B) + ({}^R \underline{\omega}_B \times \underline{H}_G) + (\underline{r}_{G/A} \times m^R \underline{a}_G) \\ &= (\underline{I}_{\underline{z}_G} \cdot {}^R \underline{\alpha}_B) + ({}^R \underline{\omega}_B \times \underline{H}_G) \end{aligned}$$



Terms in the moment equation:

$$\sum \underline{M}_A = (\underline{r}_{B/A} \times \underline{B}) + T \underline{k} = (-4\ell B_{y'}) \underline{i}' + (4\ell B_{x'}) \underline{j}' + T \underline{k}$$

$$\underline{I}_{\underline{z}_G} \cdot {}^R \underline{\alpha}_B = 2m\ell^2 \dot{\omega} \underline{i}' + (\frac{10}{3})m\ell^2 \dot{\omega} \underline{k}$$

$${}^R \underline{\omega}_B \times \underline{H}_G = \omega \underline{k} \times \underline{H}_G = 2m\ell^2 \omega^2 \underline{j}'$$

Substituting these results into the equations above gives the following

Force Equations

$$\begin{aligned} A_{x'} + B_{x'} &= 0 \\ A_{y'} + B_{y'} &= 0 \end{aligned}$$

Moment Equations

$$\begin{aligned} -4\ell B_{y'} &= 2m\ell^2 \dot{\omega} \\ 4\ell B_{x'} &= 2m\ell^2 \omega^2 \\ T &= \frac{10}{3} m\ell^2 \dot{\omega} \end{aligned}$$

Results

$$\begin{aligned} B_{x'} &= \frac{1}{2} m\ell \omega^2 = -A_{x'} \\ B_{y'} &= -\frac{1}{2} m\ell \dot{\omega} = -A_{y'} \\ T &= \frac{10}{3} m\ell^2 \dot{\omega} \end{aligned}$$