

Example 15 – Intermediate Dynamics: Bearing Loads with Misaligned Disk

Reference frames: (R is the fixed frame)

S : $\tilde{i}', \tilde{j}', \tilde{k}$ (rotates with the shaft; aligned with the shaft)

D : $\tilde{i}', \tilde{e}_2, \tilde{e}_3$ (rotates with the shaft; aligned with the disk)

Find:

$\underline{A}, \underline{B}$... the bearing loads at A and B

T ... the driving torque

Neglect weight forces.

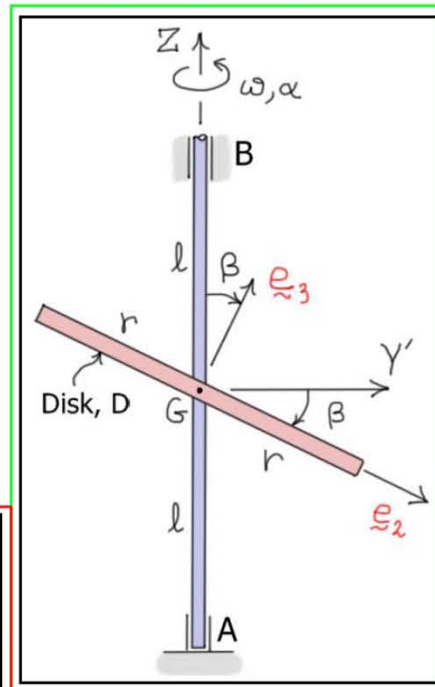
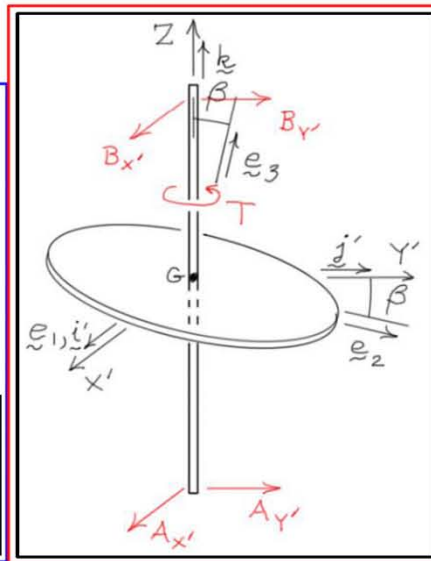
Solution:

Previous results:

$$[I_G]_D = mr^2 \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$${}^R\omega_D = \omega \tilde{k} = \omega (-S_\beta \tilde{e}_2 + C_\beta \tilde{e}_3)$$

$$\begin{aligned} \underline{H}_G &= \frac{1}{4}mr^2\omega(-S_\beta \tilde{e}_2 + 2C_\beta \tilde{e}_3) \\ &= \frac{1}{4}mr^2\omega[(S_\beta C_\beta) \tilde{j}' + (C_\beta^2 + 1)\tilde{k}] \end{aligned}$$



The *bearing loads* at A and B can be found by applying the *Newton/Euler equations of motion* to the *free-body diagram*.

$$\sum \underline{F} = \underline{A} + \underline{B} = m^R \underline{a}_G = \underline{0} \quad \Rightarrow \quad \begin{cases} A_{X'} + B_{X'} = 0 \\ A_{Y'} + B_{Y'} = 0 \end{cases} \quad (\text{scalar force equations})$$

$$\sum \underline{M}_A = (\underline{r}_{B/A} \times \underline{B}) + T \tilde{k} = (-2\ell B_{Y'}) \tilde{i}' + (2\ell B_{X'}) \tilde{j}' + T \tilde{k}$$

$$\sum \underline{M}_A = (\underline{I}_{\tilde{G}} \cdot {}^R \underline{\alpha}_B) + ({}^R \omega_B \times \underline{H}_G) = \frac{1}{4}mr^2 \dot{\omega} [(S_\beta C_\beta) \tilde{j}' + (C_\beta^2 + 1)\tilde{k}] + (-\frac{1}{4}mr^2 \omega^2 S_\beta C_\beta) \tilde{i}'$$

Scalar moment equations:

$$\begin{aligned} -2\ell B_{Y'} &= -\frac{1}{4}mr^2 \omega^2 S_\beta C_\beta \\ 2\ell B_{X'} &= \frac{1}{4}mr^2 \dot{\omega} S_\beta C_\beta \\ T &= \frac{1}{4}mr^2 \dot{\omega} (C_\beta^2 + 1) \end{aligned} \quad \Rightarrow \quad \begin{aligned} B_{X'} &= \frac{1}{8\ell} mr^2 \dot{\omega} S_\beta C_\beta = -A_{X'} \\ B_{Y'} &= \frac{1}{8\ell} mr^2 \omega^2 S_\beta C_\beta = -A_{Y'} \end{aligned}$$