

Example #16 – Intermediate Dynamics: Newton/Euler Equations of Motion

Reference frames: (R is the fixed frame)

$$B: (\underline{e}_1, \underline{n}_2, \underline{e}_3) \dots \text{(rotates with the bar } B)$$

Find:

Given Ω is *constant*, find the *equations of motion* for bar B using the *Newton/Euler equations*.

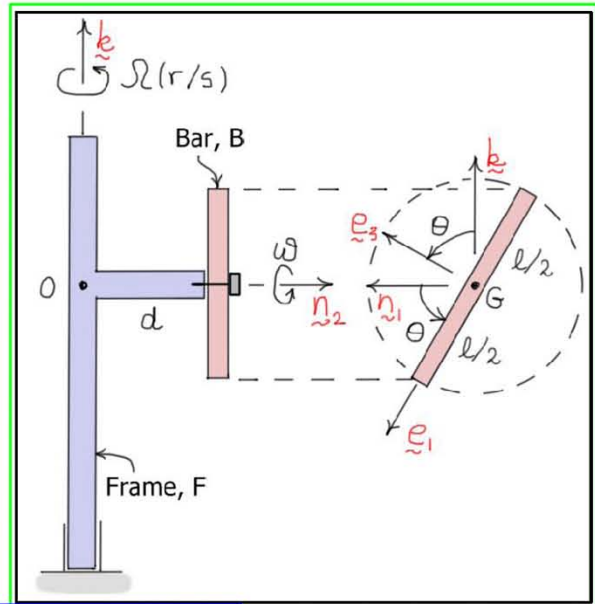
Solution:

Previous results expressed in the frame B :

$$[I_G]_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m\ell^2 & 0 \\ 0 & 0 & \frac{1}{12}m\ell^2 \end{bmatrix}$$

$${}^R\omega_B = (-\Omega S_\theta)\underline{e}_1 + \omega \underline{n}_2 + (\Omega C_\theta)\underline{e}_3$$

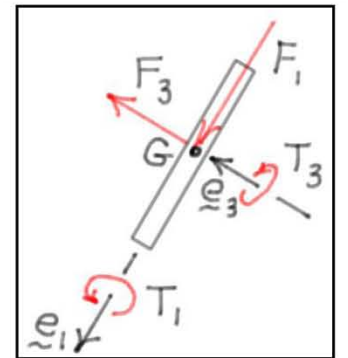
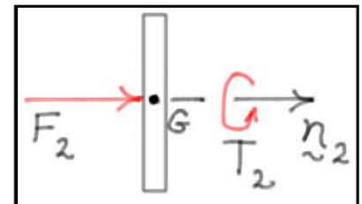
$$\underline{H}_G = \frac{1}{12}m\ell^2 (\omega \underline{n}_2 + \Omega C_\theta \underline{e}_3)$$



The equations of motion of B can be found using the Newton/Euler equations along with the *free-body diagrams* shown at the right.

$$\sum \underline{F} = F_1 \underline{e}_1 + F_2 \underline{n}_2 + F_3 \underline{e}_3 = m {}^R a_G = -m d \Omega^2 \underline{n}_2$$

$$\sum \underline{M}_G = T_1 \underline{e}_1 + T_2 \underline{e}_2 + T_3 \underline{e}_3 = (\underline{I}_G \cdot {}^R \alpha_B) + ({}^R \omega_B \times \underline{H}_G)$$



The terms on the right side of the moment equation are

$${}^R \omega_B \times \underline{H}_G = \frac{m\ell^2}{12} \begin{vmatrix} \underline{e}_1 & \underline{n}_2 & \underline{e}_3 \\ -\Omega S_\theta & \omega & \Omega C_\theta \\ 0 & \omega & \Omega C_\theta \end{vmatrix} = \frac{m\ell^2}{12} (\Omega^2 S_\theta C_\theta \underline{n}_2 - \omega \Omega S_\theta \underline{e}_3)$$

$$\underline{I}_G \cdot {}^R \alpha_B = \frac{m\ell^2}{12} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -\omega \Omega C_\theta \\ \dot{\omega} \\ -\omega \Omega S_\theta \end{Bmatrix} = \frac{m\ell^2}{12} (\dot{\omega} \underline{n}_2 - \omega \Omega S_\theta \underline{e}_3)$$

Inverse Dynamics
(Ω and ω are constant)

$$\begin{aligned} F_1 = F_3 = 0; \quad F_2 = -md\Omega^2 \\ T_1 = 0; \quad T_2 = \frac{1}{12}m\ell^2 \Omega^2 S_\theta C_\theta \\ T_3 = -\frac{1}{6}m\ell^2 \omega \Omega S_\theta \end{aligned}$$

Forward Dynamics for B
($\Omega = \text{constant}$, $\omega = \dot{\theta}$, and $\dot{\omega} = \ddot{\theta}$)

$$\begin{aligned} F_1 = F_3 = 0; \quad F_2 = -md\Omega^2 \\ T_1 = 0; \quad T_3 = -\frac{1}{6}m\ell^2 \omega \Omega S_\theta \\ \ddot{\theta} + \Omega^2 S_\theta C_\theta = 12T_2/m\ell^2 \end{aligned}$$

Equilibrium
Positions for the Bar
($T_2(t) \equiv 0$)

$$\Omega^2 S_\theta C_\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{2}$$