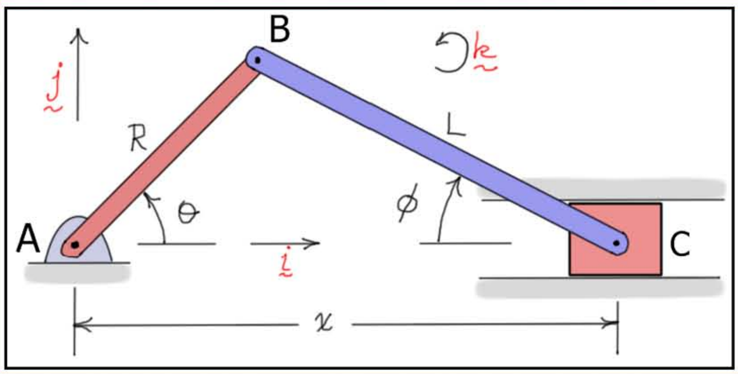
# Example #17 – Intermediate Dynamics: Partial Velocities of a Slider-Crank Mechanism

Generalized Coordinates: (not independent)

$$(q_i) = (\theta, \phi, x)$$




Find:

***Partial angular velocities*** of the links

***Partial velocities*** of the slider

Solution:

Constraint Equations and Differentiated Constraint Equations: (No offset)

$$\begin{aligned} RS_\theta - LS_\phi &= 0 \\ RC_\theta + LC_\phi - x &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} R\dot{\theta}C_\theta - L\dot{\phi}C_\phi &= 0 \\ R\dot{\theta}S_\theta + L\dot{\phi}S_\phi + \dot{x} &= 0 \end{aligned}$$

Angular Velocities of Links and Velocity of Piston:

$$\underset{\sim}{\omega}_{AB} = \dot{\theta}\,\underset{\sim}{k} = \left(\frac{LC_\phi}{RC_\theta}\right)\dot{\phi}\,\underset{\sim}{k} \qquad \underset{\sim}{\omega}_{BC} = -\dot{\phi}\,\underset{\sim}{k} = -\left(\frac{RC_\theta}{LC_\phi}\right)\dot{\theta}\,\underset{\sim}{k} \qquad \underset{\sim}{v}_C = \dot{x}\,\underset{\sim}{i} = -\left(R\dot{\theta}S_\theta + L\dot{\phi}S_\phi\right)\underset{\sim}{i}$$

Partial Angular Velocities of the Links:

$$\frac{\partial \underset{\sim}{\omega}_{AB}}{\partial \dot{\theta}} = \underset{\sim}{k} \qquad \frac{\partial \underset{\sim}{\omega}_{AB}}{\partial \dot{\phi}} = \left(\frac{LC_\phi}{RC_\theta}\right)\underset{\sim}{k} \qquad \frac{\partial \underset{\sim}{\omega}_{BC}}{\partial \dot{\phi}} = -\underset{\sim}{k} \qquad \frac{\partial \underset{\sim}{\omega}_{BC}}{\partial \dot{\theta}} = -\left(\frac{RC_\theta}{LC_\phi}\right)\underset{\sim}{k}$$

Partial Velocities of the Slider:

$$\frac{\partial \underset{\sim}{v}_C}{\partial \dot{x}} = \underset{\sim}{i}$$

$$\frac{\partial \underset{\sim}{v}_C}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}}\left[-\left(R\dot{\theta}S_\theta + \cancel{L}S_\phi\left(\frac{RC_\theta}{\cancel{L}C_\phi}\right)\dot{\theta}\right)\underset{\sim}{i}\right] = -R\left[\frac{S_\theta C_\phi + C_\theta S_\phi}{C_\phi}\right]\underset{\sim}{i} = -R\left[\frac{S_{\theta+\phi}}{C_\phi}\right]\underset{\sim}{i}$$

$$\frac{\partial \underset{\sim}{v}_C}{\partial \dot{\phi}} = \frac{\partial}{\partial \dot{\phi}}\left[-\left(\cancel{R}S_\theta\left(\frac{LC_\phi}{\cancel{R}C_\theta}\right)\dot{\phi} + L\dot{\phi}S_\phi\right)\underset{\sim}{i}\right] = -L\left[\frac{C_\phi S_\theta + S_\phi C_\theta}{C_\theta}\right]\underset{\sim}{i} = -L\left[\frac{S_{\theta+\phi}}{C_\theta}\right]\underset{\sim}{i}$$

Results at $\theta = \left\{0, \frac{\pi}{2}, \pi\right\}$:

$$\frac{\partial \underset{\sim}{v}_C}{\partial \dot{\theta}} = \begin{cases} \underset{\sim}{0} & \text{at } \theta = 0 \\ -R\,\underset{\sim}{i} & \text{at } \theta = \frac{\pi}{2} \end{cases} \qquad \frac{\partial \underset{\sim}{v}_C}{\partial \dot{\phi}} = \begin{cases} \underset{\sim}{0} & \text{at } \theta = 0 \\ \text{undefined} & \text{at } \theta = \frac{\pi}{2} \end{cases}$$