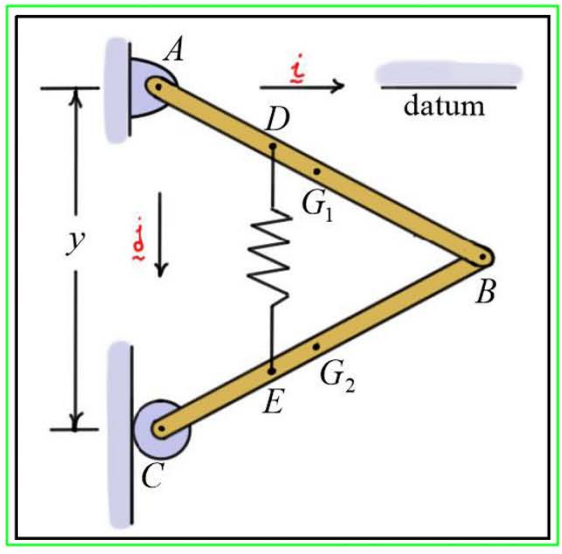
Example #19 – Intermediate Dynamics: Principle of Virtual Work (Simple Mechanism)

Given:

$$L_{AD} = L_{CE} = 0.2 \text{ (m)}; \quad L_{DB} = L_{EB} = 0.4 \text{ (m)}$$



Active Forces:

Weight forces: $m_{AB} = m_{CB} = m = 4 \text{ (kg)}$

Spring: $k = 800 \text{ (N/m)}; \ \ell_u = 0.15 \text{ (m)}$

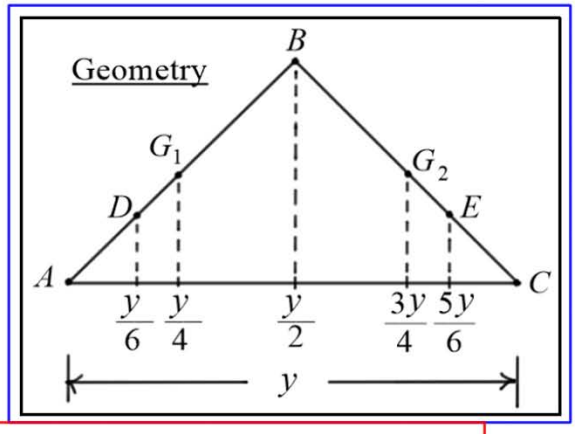
Find:

y_{EQ} the equilibrium position of C relative to A

Solution: (using y as the ***generalized coordinate***)

For equilibrium, the ***principle of virtual work*** states

$$F_y = \left(F_y\right)_{W_1} + \left(F_y\right)_{W_2} + \left(F_y\right)_{\text{spring}} = 0$$



where

$$\left(F_y\right)_{W_1} = -\frac{\partial V_1}{\partial y} = -\frac{\partial}{\partial y}\left(-mg\left(\tfrac{1}{4}y\right)\right) = \tfrac{1}{4}mg$$

$$\left(F_y\right)_{W_2} = -\frac{\partial V_2}{\partial y} = -\frac{\partial}{\partial y}\left(-mg\left(\tfrac{3}{4}y\right)\right) = \tfrac{3}{4}mg$$

$$\left(F_y\right)_{\text{spring}} = -\frac{\partial V_{\text{spring}}}{\partial y} = -\frac{\partial}{\partial y}\left(\tfrac{1}{2}k\left(\tfrac{2}{3}y - 0.15\right)^2\right) = -\tfrac{1}{2}k(2)\left(\tfrac{2}{3}y - 0.15\right)\left(\tfrac{2}{3}\right) = -\tfrac{2}{3}k\left(\tfrac{2}{3}y - 0.15\right)$$

Substituting into the principle of virtual work gives

$$F_y = \tfrac{1}{4}mg + \tfrac{3}{4}mg - \tfrac{2}{3}k\left(\tfrac{2}{3}y_{EQ} - 0.15\right) = mg - \tfrac{2}{3}k\left(\tfrac{2}{3}y_{EQ} - 0.15\right) = 0$$

$$\Rightarrow \boxed{mg = \tfrac{2}{3}k\left(\tfrac{2}{3}y_{EQ} - 0.15\right)} \quad \Rightarrow \boxed{y_{EQ} = \tfrac{3}{2}\left(\left(\frac{3mg}{2k}\right) + 0.15\right) \approx 0.335 \text{ (m)}}$$