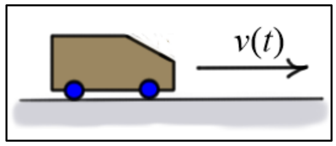
**Elementary Engineering Mathematics**
**Application of Lines in Elementary Dynamics**

Example #1




Given: Consider a car moving with ***velocity*** $v(t)$. For a ***constant braking force***, the velocity of the car satisfies the equation:

$$\boxed{v(t) = v_0 + a_0 t} \qquad (1)$$

Here, $v_0$ is the ***velocity*** of the car at the time the ***brakes*** are ***applied***, $a_0$ is the ***constant acceleration*** of the car until it stops, and $t$ is the time. During a ***test*** of the car's braking system, the following data were measured:

| Time, $t$ (s) | Velocity, $v(t)$ (ft/s) | Velocity, $v(t)$ (mi/hr) |
|---|---|---|
| 2.9 | 74.5 | 50.8 |
| 7.2 | 30.2 | 20.6 |

Find: a) $a_0$ the ***constant acceleration*** of the car; b) $v_0$ the ***initial velocity*** of the car; and c) $t^*$ the ***time required*** for the car to ***stop***. Assume a ***constant breaking force*** is applied.

Solution:

Equation (1) is in the ***slope-intercept form*** of the equation for a line: $\boxed{y = mx + b}$. Here, the ***slope*** of the line is $m = a_0$ and the $y$-intercept is $b = v_0$.

a) The slope $a_0$ can be ***estimated*** using the ***recorded data***.

$$\boxed{a_0 = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{30.2 - 74.5}{7.2 - 2.9} \approx -10.30 \text{ (ft/s}^2\text{)}}$$

Note: The symbol "$\approx$" is used here to indicate an ***approximate*** value.

So, we now have

$$\boxed{v(t) = -10.30t + v_0}$$

b) The $y$-intercept $v_0$ can now be found by using the ***slope*** and ***either*** of the ***two data pairs***.

$$v(t)\big|_{t=2.9} = 74.5 = -(10.30 \times 2.9) + v_0 \quad \Rightarrow \quad \boxed{v_0 = 74.5 + (10.30 \times 2.9) \approx 104.4 \text{ (ft/s)}}$$

or

$$v(t)\big|_{t=7.2} = 30.2 = -(10.30 \times 7.2) + v_0 \quad \Rightarrow \quad \boxed{v_0 = 30.2 + (10.30 \times 7.2) \approx 104.4 \text{ (ft/s)}}$$

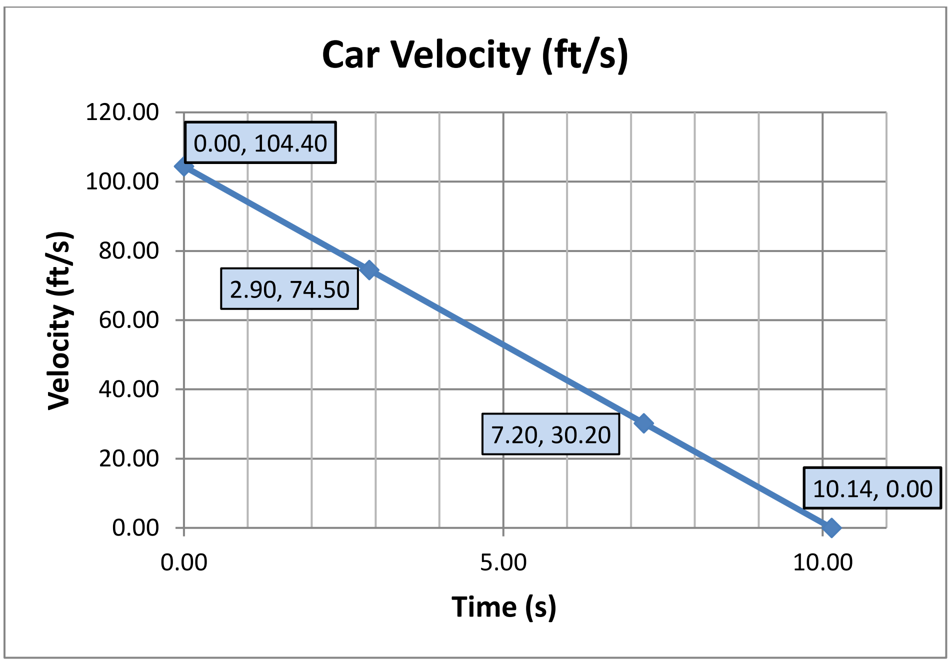
We now have the ***completed velocity equation***: $\boxed{v(t) = -10.30t + 104.4}$ (ft/s) (2)

c) Using equation (2), we can find the time $t^*$ required for the car to ***stop***.

$$v(t^*) = 0 = 104.4 - (10.30t^*) \quad \Rightarrow \quad \boxed{t^* = 104.4 / 10.30 \approx 10.14\ (s)}$$

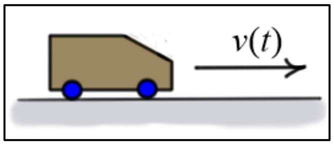
<u>Note</u>:

The ***stopping time*** $t^*$ and the ***initial velocity*** $v_0$ are the ***x***- and ***y-intercepts*** of the line.



Example #2

Given: Again, consider a car moving with ***velocity*** $v(t)$. As before, for a ***constant braking force***, the velocity of the car satisfies the equation:




$$\boxed{v(t) = v_0 + a_0\,t} \qquad (3)$$

During a ***second test*** of the car's braking system, the following data were measured:

| Time, $t$ (s) | Velocity, $v(t)$ (ft/s) | Acceleration, $a$ (ft/s$^2$) |
|---|---|---|
| 4.3 | 59.7 | –10.5 |

Find: a) $v_0$ the ***initial velocity*** of the car; and b) $t^*$ the time required for the car to ***stop***. Assume a ***constant breaking force*** is applied.

Solution:

a) To find the ***initial velocity*** $v_0$, we can use the ***point-slope form*** of the equation for a line.

$$\frac{y - y_1}{x - x_1} = m \quad \Rightarrow \quad \frac{v(t) - 59.7}{t - 4.3} = -10.5 \quad \Rightarrow \quad v(t) - 59.7 = -10.5(t - 4.3)$$

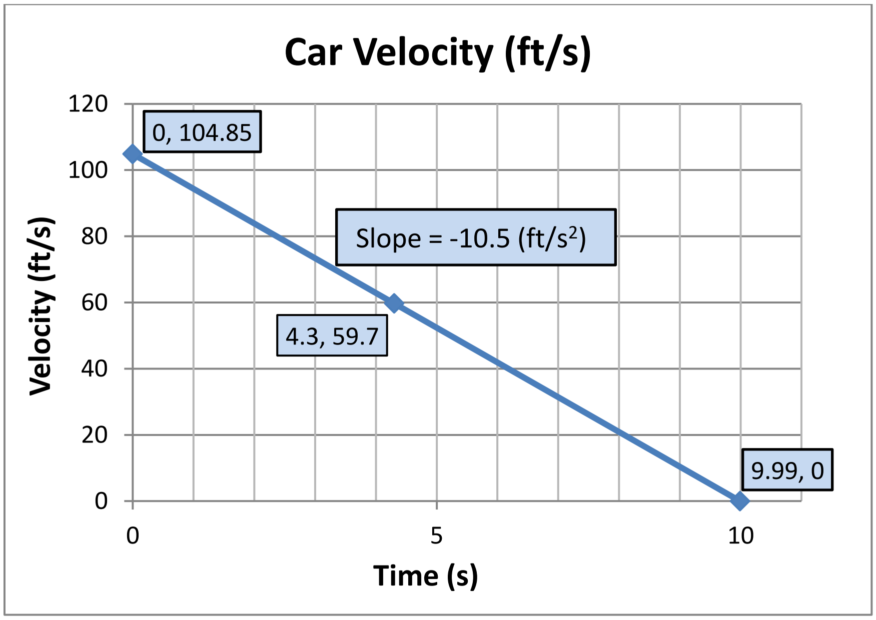
or

$$v(t) = \big(59.7 + (10.5 \times 4.3)\big) - 10.5t \quad \Rightarrow \quad \boxed{v(t) = 104.85 - 10.5t} \text{ (ft/s)} \qquad (4)$$

Comparing equations (3) and (4) yields: $\boxed{v_0 = 104.85 \text{ (ft/s)} \approx 71.5 \text{ (mi/hr)}}$

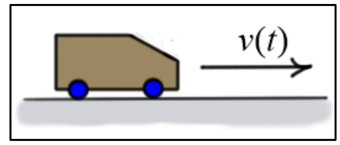
b) Equation (4) can also be used to find the time $t^*$

$$v(t^*) = 0 = 104.85 - (10.5\, t^*) \quad \Rightarrow \quad \boxed{t^* = 104.85 / 10.5 \approx 9.99 \text{ (s)}}$$



Example #3

Given: Now, consider a car that ***starts*** at ***rest***, ***accelerates*** at a ***constant rate*** of $a_0 = 14.8\ (\text{ft/s}^2)$ for 6 seconds, and then ***decelerates*** at a ***constant rate*** $a_1 = -10.5\ (\text{ft/s}^2)$ until it ***stops***. Since the car ***starts*** from ***rest***, during the ***constant acceleration phase***, the velocity of the car satisfies the equation




$$\boxed{v(t) = a_0\, t = 14.8\, t} \qquad (5)$$

During the ***constant deceleration phase***, the velocity of the car satisfies the equation

$$\boxed{\frac{v(t) - v(t)\big|_{t=6}}{t-6} = a_1 = -10.5} \qquad \text{(Point-slope form)} \qquad (6)$$

Find: a) the equation for $v(t)$ that applies during the ***deceleration*** phase, and b) $t^*$ the time when the car stops.

Solution:

a) Using equation (5), we find the velocity of the car at $t = 6\,(\text{sec})$ to be

$$\boxed{v(t)\big|_{t=6} = a_0\, t\big|_{t=6} = (14.8)(6) = 88.8\ (\text{ft/s})}$$

Substituting into the point-slope form in equation (6) and reorganizing terms gives

$$\frac{v(t) - v(t)\big|_{t=6}}{t-6} = \frac{v(t) - 88.8}{t-6} = -10.5$$

$$\Rightarrow \quad v(t) - 88.8 = -10.5\big(t-6\big)$$

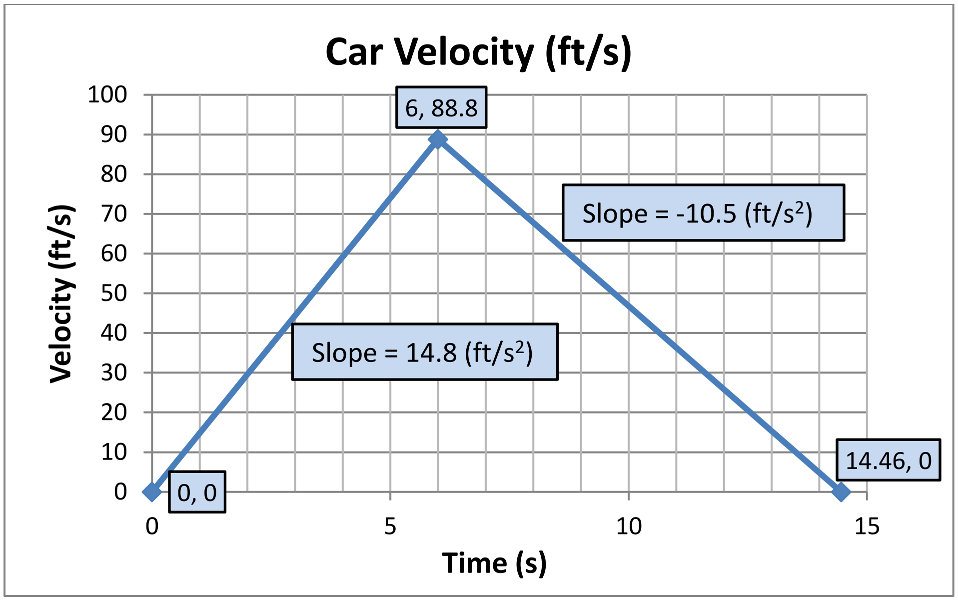
$$\Rightarrow \quad v(t) = \big(88.8 + \big(6 \times 10.5\big)\big) - 10.5t$$

or

$$\boxed{v(t) = 151.8 - 10.5t}\ (\text{ft/s})$$

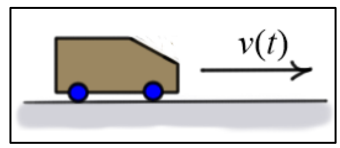
b) To find the time $t^*$ when the car stops, set

$$v(t^*) = 0 = 151.8 - (10.5t^*) \quad \Rightarrow \quad \boxed{t^* = 151.8/10.5 \approx 14.46\ (\text{s})}$$




Example #4

Given: Now, consider a car that ***starts*** at ***rest***, ***accelerates*** at a ***constant rate*** of $a_1 (\text{m/s}^2)$ for $t_1$ seconds, and then ***decelerates*** at a ***constant rate*** $a_2 (\text{m/s}^2)$ until it ***stops*** at time $t_2$. Since the car ***starts*** from ***rest***, during the ***constant acceleration phase***, the ***velocity*** of the car satisfies the equation




$$\boxed{v(t) = a_1 t} \qquad (a_1 > 0) \qquad (7)$$

During the ***constant deceleration phase***, the velocity of the car satisfies the equation

$$\boxed{\frac{v(t) - v(t)\big|_{t=t_1}}{t - t_1} = -a_2} \qquad (a_2 > 0) \qquad \text{(Point-slope form)} \qquad (8)$$

Find: a) time $t_2$ in terms of time $t_1$ and the acceleration and deceleration rates $a_1$ and $a_2$, and b) $(t_2 - t_1)/t_1$ the ratio of the time durations of deceleration and acceleration.

Solution:

a) Using the ***point-slope*** form in equation (8), and ***substituting*** for $v(t_1)$ using equation (7) gives

$$\frac{v(t) - v(t)\big|_{t=t_1}}{t - t_1} = -a_2$$

$$\Rightarrow \quad v(t) - a_1 t_1 = -a_2 (t - t_1)$$

$$\Rightarrow \quad \boxed{v(t) = (a_1 + a_2)\, t_1 - a_2 t} \ (\text{m/s})$$

Now, using the fact that $v(t_2)=0$, we can solve for the time $t_2$ as follows

$$v(t_2)=0=(a_1+a_2)t_1-a_2t_2 \quad \Rightarrow \quad \boxed{t_2=\left[\frac{a_1+a_2}{a_2}\right]t_1} \text{ (s)} \qquad (9)$$

b) Using equation (9), we can solve for the ***ratio*** of the ***time durations*** of the ***deceleration*** and ***acceleration*** phases.

$$t_2-t_1=\left[\frac{a_1+a_2}{a_2}\right]t_1-t_1=\left[\frac{a_1+a_2}{a_2}-1\right]t_1=\left[\frac{a_1+a_2}{a_2}-\frac{a_2}{a_2}\right]t_1=\left[\frac{a_1+\cancel{a_2}-\cancel{a_2}}{a_2}\right]t_1=\left[\frac{a_1}{a_2}\right]t_1$$

or

$$\boxed{\frac{t_2-t_1}{t_1}=\frac{a_1}{a_2}}$$